# COMPARING THE EFFECTIVENESS OF INQUIRY-BASED LEARNING AND DIRECT INSTRUCTION ON ENHANCING MATHEMATICAL THINKING IN SECONDARY SCHOOL STUDENTS

**SHASHA LU**

**A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS IN EDUCATION AND SOCIETY
INSTITUTE OF SCIENCE INNOVATION AND CULTURE
RAJAMANGALA UNIVERSITY OF TECHNOLOGY KRUNGTHEP
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**Major** Master of Arts (Education and Society)

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## ABSTRACT

This study aims to investigate the effect of inquiry-based learning and direct instruction on enhancing mathematical thinking abilities among secondary school students, and to examine students' attitudes towards inquiry-based learning as a method for improving mathematical thinking abilities. This study adopted the quantitative research method. Eighty-six questionnaires were issued, and 86 were valid, with a validity of 83.5%. The mathematical thinking ability test consists of pre-test and post-test items. There are 43 students from Class 1- the control group, and 43 from Class 2- the experiment group, Grade 3. This study finds that the achievements of inquiry-based learning are superior to those of direct instruction in improving secondary school students' mathematical thinking abilities. Students who learn through inquiry-based learning report a high level of satisfaction. Based on the analysis results, the following suggestions have been put forward. Teachers should cultivate students' confidence and abilities by guiding more active discussions and cooperation. Teachers should cultivate students' patience and strategic awareness in problem-solving. The success of this teaching method relies on teacher guidance, an open classroom atmosphere, and active student interaction.

**Keywords: Inquiry-Based Learning, Mathematical Thinking Abilities, Secondary School Students**

# ACKNOWLEDGEMENTS

I would like to take this opportunity to express my sincere gratitude to everyone who immensely supported me throughout the Master of Arts in Education and Society studies, especially my academic advisor, Assistant Professor Dr. Wannaporn Siripala, and the other professors who consistently helped me complete my thesis and provided me with valuable feedback and encouragement. I am thankful for their inspiring guidance, invaluably constructive criticism, and friendly advice during the project work. Furthermore, I would like to thank all the participants in my survey who willingly shared their time during the data collection process. Lastly, I would like to thank my loved ones, my family, and my friends, who have remarkably supported me throughout the entire learning process, especially my mother, who helped me complete my questionnaires. It would have been impossible to complete my thesis without the help of these people.

Shasha LU

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# CHAPTER 1

# INTRODUCTION

## 1.1 Background and Rationale

In recent years, educational and learning theories have evolved in tandem with waves of educational reform, prompting schools and teachers to reevaluate long-standing teaching and talent development practices (Gopinathan et al., 2022). With the deepening reform of mathematics education in China, inquiry-based learning in mathematics has garnered increasing attention. Since China introduced new curriculum reform standards, new demands have been placed on teachers, advocating for active interaction and mutual development between teachers and students during the teaching process. These include striking a balance between traditional knowledge and skill development (Song et al., 2017). Mathematics instruction should focus on fostering students' independence and autonomy, guiding them to question, investigate, and explore, learning through practice, and encouraging them to engage in active, personalized learning under the guidance of teachers (Guo et al., 2018). The fundamental goal is to transform learning methods and achieve quality education.

The Chinese Ministry of Education "General Middle School Mathematics Curriculum Standards" (2017 edition, revised in 2020) clearly states in its section on the nature of the curriculum that "mathematics plays an irreplaceable role in shaping rational thinking, scientific spirit, and promoting the intellectual development of individuals." (Chen et al., 2020). Mathematics is closely related to rationality; mathematical thinking abilities are characterized as an indispensable form of thinking for everyone in modern society. The middle school curriculum standards also emphasize in their educational philosophy that "advocating independent thinking, autonomous learning, cooperative communication, and other diverse learning methods stimulates interest in learning mathematics, fosters good learning habits, and promotes the development of students' practical abilities and innovative consciousness. (Chen et al., 2020) " The development of students' practical abilities and innovation is tied to the enhancement of mathematical thinking abilities. Seeking innovative and effective learning methods is not only a requirement of the curriculum philosophy but also a necessary path for improving middle school students' mathematical thinking abilities. With the new curriculum standards, mathematical core literacy has become the focus of middle school mathematics teaching (Bao et al., 2016). The traditional, singular approach to mathematics instruction, which progresses from "observation" to "analysis" to "expression," no longer meets the requirements for cultivating students' mathematical core literacy. Mathematics educators need to seek breakthroughs, adopting more diverse teaching methods that emphasize process and skill development. For students, mathematics should not be merely a process of "input" but also a process of "output" (Cui et al., 2019).

In the current stage of mathematics education in China, the teaching approach is predominantly exam-oriented, with a focus on improving students' test scores. Teachers typically lead the instruction, while students' learning behavior is reduced to simple imitation. This results in a dull and passive classroom atmosphere, leading to low teaching efficiency (Cui et al., 2019). Due to the highly abstract and logically rigorous nature of mathematics, teachers often concentrate most of their attention on the content rather than on guiding students' thinking and enhancing their learning abilities. Most teachers employ a "lecture-practice-lecture" model, aiming to improve student performance through detailed explanations and practice drills (Chen et al., 2020). However, this approach fails to encourage students to explore the processes behind formulas, rules, concepts, and principles, making it difficult for them to form a comprehensive cognitive structure. Students remain at a level of rote memorization, with limited active thinking in class, which restricts their ability to think critically. When faced with more challenging problems, they often find themselves at a loss, unable to devise solutions (Chen et al., 2020).

In addition to the monotonous teaching methods, mathematics instruction heavily relies on textbook exercises and test papers, which are not closely connected to real life. This disconnect results in a lack of awareness and ability among middle school students to apply mathematical knowledge in everyday situations (Cui et al., 2019). Essentially, mathematics reflects phenomena and problems in life; thus, weakening the connection between mathematics and life hinders the development of students' mathematical abilities, thereby affecting the formation and development of their core competencies.

As understanding in the field of education deepens, the original singular teaching objectives have gradually evolved into three-dimensional teaching objectives, positively impacting students' overall development. However, the current assessment methods used by mathematics teachers are still primarily focused on tests, with students' exam scores serving as the primary criterion for evaluation. This overly simplistic evaluation method does not align with the current trend of emphasizing the development of students' mathematical core competencies, nor does it enable students to master mathematics (Gopinathan et al., 2022). While students' math scores can provide some insight into their grasp of mathematical knowledge, relying solely on this form of assessment is one-dimensional, lacks flexibility, and fails to accurately reflect students' mathematical competence, especially their mathematical thinking abilities (Guo et al., 2018).

Therefore, every mathematics educator must consider establishing an effective learning system. Inquiry-based learning within modern education offers a new approach to learning, allowing students to re-explore, understand, and organize mathematical knowledge. Through inquiry-based learning, students gain a deeper understanding of mathematical concepts and construct their cognitive structures (Gopinathan et al., 2022; Guo et al., 2018). This process also allows mathematics teachers to gain deeper insights into students' learning, providing a new, equitable platform for dialogue between teachers and students.

## 1.2 Research Questions

In recent years, research on mathematics learning has made significant strides compared to earlier studies; however, most of this research focuses on specific aspects of the mathematics learning process. The studies concentrate on teaching practices, current status surveys, evaluation mechanisms, and implementation strategies related to the effectiveness of mathematics learning (Gopinathan et al., 2022). Moreover, the research subjects are predominantly concentrated at the elementary and middle school levels. In practice, these methods are more commonly applied in lower grades than in higher grades, with a greater emphasis on practical application than on theoretical advancement. The implementation phase of mathematics learning is the aspect that concerns mathematics teachers the most, which explains the abundance of research in this area.

Approaching the subject from the perspective of mathematics itself, one can explore its relationship with other factors and thereby seek effective strategies. As the embodiment of rational thought, mathematics represents a significant leap in the abstraction and logical complexity of concepts from the middle school level compared to earlier stages (Darlington, 2014). This progression requires middle school students to employ higher-level thinking skills to tackle more abstract and complex mathematical problems. Learning methods are crucial tools for acquiring mathematical knowledge, and they are intricately linked to mathematical thinking abilities. Therefore, investigating the relationship between these two elements contributes to students' mathematical thinking abilities.

How does inquiry-based learning compare to direct instruction in enhancing mathematical thinking abilities among secondary school students?

What are the students' attitudes towards inquiry-based learning as a method for enhancing mathematical thinking abilities in secondary school?

## 1.3 Research Hypotheses

The hypotheses are proposed:

The achievements of inquiry-based learning are better than direct instruction in improving secondary school students' mathematical thinking abilities.

Students who learn with inquiry-based learning to improve secondary school students' mathematical thinking abilities have a high level of satisfaction.

## 1.4 Research Objectives

This research illustrates inquiry-based learning in mathematics instruction through study plans, teaching tests, and surveys. It details the operational process of implementing inquiry-based learning in classroom teaching, with the expectation of providing practical guidance and specific case studies for organizing task-based inquiry learning after its application. The study aims to conduct a systematic analysis and practice of inquiry-based learning instructional design, focusing on its impact on developing students' mathematical thinking abilities. Solving mathematical problems requires students to possess strong logical thinking, abstract reasoning, and mathematical creativity, as well as a broad and in-depth understanding of mathematical concepts and principles. Moreover, mathematical problem-solving is a crucial pathway for cultivating students' mathematical abilities and logical thinking skills.

To investigate the effect of inquiry-based learning and direct instruction in enhancing mathematical thinking abilities among secondary school students.

To examine students' attitudes towards inquiry-based learning as a method for improving mathematical thinking abilities in secondary school.

## 1.5 Scope and Limitation of the Research Study

### 1.5.1 Scope of the Research Study

This research focuses on examining the impact of direct instruction and inquiry-based learning on enhancing middle school students' mathematical thinking abilities. The study provides a thorough analysis of the roles of these two teaching methods in enhancing students' logical thinking, problem-solving abilities, and abstract thinking. By comparing these instructional approaches, the research aims to reveal differences in their effectiveness in cultivating these skills. Additionally, the study explores students' acceptance and satisfaction with inquiry-based learning, including their feedback and perceptions after participating in this learning method. This evaluation determines whether inquiry-based learning aligns with students' needs, stimulates their interest in mathematics, and effectively fosters the development of their mathematical thinking abilities. Considering the characteristics of inquiry-based learning and the varying learning tasks at different stages of middle school, the research involves third-year students from Guang Ming Middle School, specifically from Class (1) and Class (2). After surveying to assess the current state of students' mathematical learning and thinking abilities, a mathematics learning plan was developed based on the findings, along with the textbooks used and the review schedule, followed by a teaching experiment.

### 1.5.2 Limitations of the Research Study

The limitations of this research include the generalizability and applicability of its findings. Although the study selects a representative sample of middle school students, differences in geographic, cultural, or socioeconomic backgrounds may limit the applicability of the results to all middle school students. The limited sample size may introduce bias into the statistical analysis, affecting the reliability of the findings. Additionally, the study's timeframe may constrain its conclusions, as a shorter duration might not be sufficient to observe the more profound impact of inquiry-based learning and direct instruction on the development of students' mathematical thinking abilities.

Student-related factors present another significant limitation. Variations in students' motivation, interest in learning, and individual study habits can affect the effectiveness of teaching methods, and these subjective factors are challenging to control within the research design. This means that even under identical teaching conditions, the improvement in students' mathematical thinking abilities may vary significantly. The research also faces limitations related to sample selection, study duration, teacher variability, and the definition and implementation of teaching methods. These factors should be carefully considered when interpreting the results and addressed in future research to optimize outcomes.

## 1.6 Research Framework

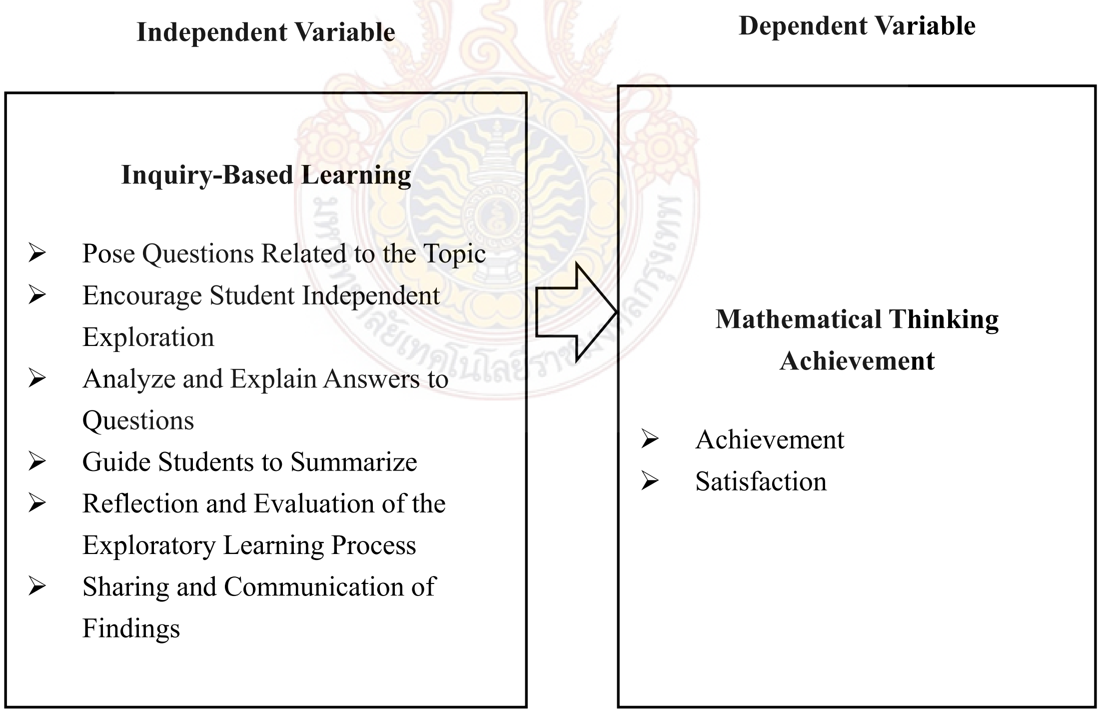


Figure 1.1 Conceptual Framework

## 1.7 Definition of Key Terms

### 1.7.1 Inquiry-Based Learning

Inquiry-Based Learning (IBL) is a student-centered teaching approach that fosters in-depth understanding and critical thinking. It guides students to ask questions, conduct investigations, analyze information, and draw conclusions.

The steps of inquiry-based learning:

(1) Pose Questions Related to the Topic

Encourage students to ask meaningful questions about the topic to spark curiosity and establish the focus of their inquiry.

(2) Encourage Student Independent Exploration

Guide students to gather information independently, design experiments, and solve problems, fostering autonomy and critical thinking.

(3) Analyze and Explain Answers to Questions

Help students organize and analyze the data they have collected, allowing them to interpret findings and form initial conclusions.

(4) Guide Students to Summarize

Assist students in summarizing their findings, making connections, and consolidating their understanding of the topic.

(5) Reflection and Evaluation of the Exploratory Learning Process

Encourage students to reflect on their learning process, evaluate their methods and results, and identify areas for improvement.

(6) Sharing and Communication of Findings

Provide opportunities for students to share their findings through presentations, discussions, or reports, enhancing collaboration and knowledge sharing.

### 1.7.2 Mathematical Thinking Abilities

Mathematical thinking abilities share characteristics with general thinking, such as abstraction, indirectness, and generalization. However, the scope of mathematical thinking differs from general thinking abilities, as it also encompasses the understanding, internalization, and application of quantitative relationships, spatial forms, mathematical structures, and more. Therefore, mathematical thinking abilities possess their distinct characteristics.

**1.7.3 Secondary School Students**

Secondary school students are those receiving education at the secondary level. Depending on the education system of different countries and regions, secondary education typically refers to the middle school stage, generally encompassing students aged 14 to 16.

**1.7.4 Satisfaction**

Satisfaction refers to the degree to which students are content and feel positive about the learning process. It can be measured using a survey or questionnaire that asks students to rate their overall experience, enjoyment, engagement, and perceived value of the learning activities.

**1.7.5 Achievement**

Achievement refers to the measurable improvement in students' mathematical thinking abilities resulting from the instructional method used (IBL or Direct Instruction).

**1.7.6 Direct Instruction**

Direct instruction refers to a teacher-centered, highly structured, and systematic approach to instruction, where the teacher explicitly defines learning objectives and methodically delivers content through step-by-step explanations and demonstrations.

# CHAPTER II

# LITERATURE REVIEW

## 2.1 Related Theories of Inquiry-Based Learning

### 2.1.1 Constructivist Theory

Constructivist theory, also known as structuralism theory, originates from the theory of cognitive development in children and is linked to cognitive development. Constructivists believe knowledge cannot be directly transmitted to learners through simple instruction; instead, it must be actively acquired and internalized through participation in social and cultural activities (Watson, 2011). Internalization is central to constructivist theory, describing how individuals connect and integrate new information with their existing knowledge structures upon receiving it.

The roots of constructivist theory can be traced back to Piaget's theory of cognitive development in children (Schmidt, 1987). According to Piaget, children's cognitive development is achieved through continuous interaction with the environment and the absorption and integration of new information during this process (Schmidt, 1987). Children’s thinking is dynamic and constantly evolving; they gradually build more complex cognitive structures by adapting to new environments and information. Constructivists draw from this idea, asserting that knowledge acquisition is not the result of passive reception but an active construction (Zander, 2010). When faced with new information, learners first attempt to assimilate it into their existing cognitive structures—a process known as assimilation. However, when new information does not fully align with existing knowledge structures, learners must adjust or expand cognitive structures to better integrate the knowledge. This adjustment process is referred to as accommodation (Tay, 2016).

In constructivist theory, learning is viewed as a process of social interaction and construction. Learners acquire knowledge through interacting with others and participating in social and cultural activities, then internalize this knowledge into their understanding through reflection and discussion (Zhang & Liu, 2014). This process involves more than just mechanical memorization or imitation; it requires dynamic interaction with the surrounding environment, continuous adjustment, verification, and reconstruction of cognitive structures, leading to a profound understanding and mastery of knowledge. In educational settings, constructivist theory emphasizes that teachers are not mere transmitters of knowledge but rather guides or facilitators of learning (Prawat & Floden, 1994). Teachers design meaningful learning tasks, encourage students to engage in discussions, cooperation, and inquiry, and help them construct knowledge while solving real-world problems. In this process, students are not passive recipients of information but actively explore, think, and practice, gradually building their understanding of the content (Prawat & Floden, 1994).

Constructivism emphasizes the contextual nature of learning, asserting that the acquisition of knowledge is closely tied to the actual social and cultural contexts in which it occurs. This means that knowledge is not merely an abstract concept or theory but is closely related to concrete practices and applications (Watson, 2000). Learners can better understand the meaning and value of knowledge and transform it into practical skills by applying it in real-life situations. Constructivist theory suggests that the acquisition and understanding of knowledge are active, dynamic, and contextualized processes (Deibl et al., 2018). Learners integrate and adjust new information with existing knowledge structures through interaction with the environment, thereby achieving cognitive development. This theory has profoundly influenced educational practice, driving from teacher-centered models to student-centered learning models, emphasizing learners' active participation in knowledge construction (Fong et al., 2016).

Constructive learning involves learners continuously comparing new and existing knowledge to form and adjust their cognitive structures. Based on constructivist theory, learners must build their knowledge structures (Deibl et al., 2018).

In the process of constructive learning, the teacher acts as a guide for student learning, while students are the main subjects of learning. Teachers lead students in constructing their knowledge systems, and students actively select, process, and interpret external knowledge from teachers or other sources, then recode and reinterpret the newly acquired knowledge, integrating it into their personal knowledge system to form their understanding (Deibl et al., 2018).

**2.1.2 Cognitive Assimilation Theory**

Cognitive psychology is a branch of psychology that studies cognitive processes from an information-processing perspective (Tian et al., 2019). Cognitive psychologists infer unobservable mental processes based on observable phenomena, providing unique insights into education. Knowledge is considered not to be predetermined within the cognitive subject but is primarily acquired through external integration. When children incorporate relevant information from the external environment into their existing cognitive structures (also known as "schemas"), this process is referred to as "assimilation." If new knowledge cannot be assimilated, it triggers "accommodation," a process in which changes in the external environment lead to the modification and reorganization of existing cognitive structures when they can no longer accommodate the new information (Nang et al., 2022). Assimilation and accommodation are the two fundamental processes by which students perceive and understand the world. Assimilation leads to the expansion of cognitive structures (i.e., schema expansion), while accommodation results in changes to the nature of cognitive structures (i.e., schema modification). Children's cognitive structures are gradually constructed through the balance achieved between assimilation and accommodation in their interactions with the environment (Shogren et al., 2006).

According to genetic epistemologists, knowledge arises from continuous construction, and each act of understanding by the learner involves some degree of invention (Koufaris, 2022). The transition from one developmental stage to another is characterized by the emergence of new structures that previously did not exist in the external world or the subject's cognitive framework. When students learn new knowledge, with the help of the teacher's guidance and organization, they attempt to absorb the new knowledge from different perspectives by utilizing their existing knowledge, ultimately incorporating it into their cognitive structures as their knowledge (Nang et al., 2022). Cognitive assimilation theory has had a profound impact on academia. The new knowledge that students learn should have "potential meaning," and the content of new learning materials should be able to establish substantive, non-arbitrary connections with the relevant knowledge in the students' existing cognitive structures. Furthermore, students should have a "predisposition" for meaningful learning, proactively and actively retrieving appropriate schemas from their existing knowledge structures to "assimilate" new materials and make adjustments to reconcile discrepancies and conflicts between old and new knowledge (Koufaris, 2022). Learning needs to transform "potentially meaningful learning materials" into organic components of the subject's knowledge structure, thereby promoting cognitive development in students.

The function of teaching is to facilitate student development, and authentic tasks are more likely to stimulate student learning. Cognitive assimilation theory places great importance on the tasks that students encounter in their learning (Ritter et al., 2017). Through learning tasks, students become aware that their current level of development is insufficient to complete the tasks, requiring guidance from teachers or assistance to promote development (Nang et al., 2022). When learners recognize their knowledge deficiencies, it fosters active thinking and a willingness to seek help, making learning more proactive and inquiry-based. Scholars of cognitive assimilation theory believe that one of the main functions of school education is to stimulate students' interest in learning by creating cognitive conflicts or incomplete cognitive systems, motivating learners to work towards resolving them. The stimuli that cause cognitive conflict originate from learning tasks (Fong et al., 2016). Students can engage in classroom learning in a manner that closely resembles real-life situations, incorporating the knowledge presented by the teacher into their cognitive systems through active thinking, hands-on activities, and the summarization and organization of information (Fong et al., 2016; Tian et al., 2019). In this process, students personalize their interpretation of knowledge while experiencing the practical connection between learning and life.

## 2.2 Related Studies

### 2.2.1 Inquiry-Based Learning and Steps to Teach

#### *2.2.1.1 The Concept of Inquiry-Based Learning*

Inquiry-Based Learning (IBL) was advocated by Professor Schwab of the University of Chicago in the 1950s as part of the "Modernization of Education" movement (Rasi, 2015). Currently, there is no unified definition of IBL. Aparicio-Ting et al. (2019) focus on a phased approach to Inquiry-Based Learning (IBL) as a driver of curriculum. Their research found that IBL effectively encourages students to actively engage in the process of acquiring knowledge, develop the inquiry skills necessary for studying nature, and form scientific concepts, ultimately fostering a proactive attitude towards exploring the world. Meanwhile, Oliver et al. (2019) explored the perceptions and practical application of IBL among science teachers in the UK and Spain. Their research indicated that although IBL is highly recommended in education, its actual use in science education in the UK and Spain is relatively limited. The study further revealed the obstacles and challenges teachers face in implementing IBL, highlighting the need for greater support to enable teachers to apply this teaching method effectively. Inquiry-based learning typically involves selecting and defining research topics from academic disciplines or real-life scenarios, creating contexts for students (Aparicio-Ting et al., 2019). This approach aims to enhance students' knowledge, skills, emotional attitudes, and spirit of inquiry and innovation. Mathematics learning for elementary students should not merely involve mastering and practicing concepts, rules, and formulas, but should also encourage inquiry and critical thinking. Aditomo et al. (2013) explored the principal forms, educational objectives, and disciplinary variations of Inquiry-Based Learning (IBL) in higher education. The research found that IBL can effectively stimulate students' curiosity and cultivate their spirit of inquiry. The study points out that children are naturally curious and possess a strong spirit of inquiry. Nature is characterized by activity, expression, and creativity, and these traits are fully developed and expressed through inquiry-based learning.

Ibrohim et al. (2020) conducted a study on the implementation of Inquiry-Based Learning (IBL) to enhance students' understanding of the nature of science (NOS). Their research found that mathematical inquiry-based learning primarily involves students applying their existing knowledge and life experiences under the guidance of teachers to tackle challenging problems or tasks. This approach not only deepens students' understanding of scientific concepts but also fosters critical thinking and problem-solving skills. Through independent learning and collaborative discussion, students acquire knowledge, apply it, and solve problems. Inquiry activities help students achieve a deeper understanding of mathematical concepts, learn general methods for discovering and understanding mathematics, and cultivate a spirit of exploration and innovation.

Inquiry-Based Learning (IBL) is an educational approach that encourages students to actively engage in the learning process by applying their existing knowledge and life experiences to tackle challenging problems or tasks. By guiding students through independent learning and collaborative discussions, IBL deepens their understanding of scientific and mathematical concepts, enhances their inquiry skills, and promotes innovation.

***2.2.1.2 Step of Inquiry-Based Learning***

Inquiry-based learning is characterized by openness, involving the interplay and disorder between learning content and individuals' experiences and knowledge. This disorder stimulates students' interest in organizing and making sense of information, driving their desire to acquire knowledge (Aditomo et al., 2013; Aparicio-Ting et al., 2019; Rasi, 2015). Effective inquiry-based learning requires teachers to actively regulate students' problem-solving abilities, potential inquiry directions, and emotional engagement. The knowledge involved in inquiry-based learning often spans multiple disciplines. It is more closely tied to real-life and social practices compared to traditional knowledge acquisition, thereby contributing to students' practical skills (Capps & Crawford, 2013).

(1) Pose Questions Related to the Topic

The core of inquiry-based learning is to encourage students to ask meaningful questions. These questions are typically open-ended and exploratory, requiring students to engage in thinking, reasoning, and investigation beyond simple recall or reproduction (Rasi, 2015). Teachers play a guiding role at this stage, helping students formulate challenging and worthwhile questions.

(2) Encourage Student Independent Exploration

After identifying a problem, students enter the investigation and research phase. They need to collect data and information relevant to the problem, which may include experiments, field studies, literature reviews, or consultations with experts. This process emphasizes the development of students' self-directed learning skills and information literacy, encouraging them to critically analyze the information they gather (Oliver et al., 2019).

(3) Analyze and Explain Answers to Questions

Once sufficient information is collected, students analyze and interpret the data. This step requires students to connect their findings with existing knowledge, analyze the meaning of the information, and draw preliminary conclusions through reasoning. This phase develops students' logical thinking and problem-solving abilities (Capps & Crawford, 2013).

(4) Guide Students to Summarize

Based on the analysis and interpretation, students should be able to conclude. These conclusions should address the initial questions and be supported by data and analysis. Forming conclusions not only reflects the learning outcomes but also demonstrates students' understanding of the inquiry process.

(5) Reflection and Evaluation of the Exploratory Learning Process

Reflection is a crucial component of inquiry-based learning. Students need to reflect on their learning process, including the methods of questioning, the effectiveness of their investigations, the rigor of their analysis, and the reliability of their conclusions. Both teachers and students evaluate the entire inquiry process to identify successes and areas for improvement.

(6) Sharing and Communication of Findings

Finally, inquiry-based learning encourages students to share their findings and conclusions. Through presentations, reports, discussions, and other forms of communication, students present their learning outcomes to peers, teachers, or a broader audience. This phase not only helps consolidate students' learning but also develops their communication skills and teamwork.

***2.2.1.3 The Significance of Inquiry-Based Learning***

In the information society, learning must be integrated into productive factors. Education must take on the responsibility of improving students' learning methods. Education is no longer merely about transmitting textbook knowledge and skills. New curricula are calling for, anticipating, and directly driving changes in learning methods (Little, 2008). This transformation in learning methods is a focal point of classroom teaching reform and a highlight of curriculum reform. However, changing learning methods first requires a transformation in teaching methods. Teachers must create conditions for students to engage in autonomous, collaborative, and inquiry-based learning (Buckner & Kim, 2013). They must provide time and space for active learning, thinking, and experiential learning, freeing students from a singular reliance on textbooks and closed classroom environments. Instead, learning should occur through activities and real-life experiences, utilizing and integrating various teaching resources. Implementing such changes requires research and adaptation by frontline educators; otherwise, reforming learning methods remains unachievable. Transforming learning methods is a requirement of social informatization, economic globalization, and the knowledge economy (Aparicio-Ting et al., 2019). Adapting and improving learning methods helps establish autonomous, collaborative, and inquiry-based learning approaches, which affect not only the learning itself but also students' mental states.

The revised mathematics curriculum standards explicitly state that teachers should stimulate students' enthusiasm for learning, provide ample opportunities for engaging in mathematical activities, and help students understand and master fundamental mathematical knowledge, skills, ideas, and methods through independent exploration and collaborative interaction (Oliver et al., 2019). Mathematics inquiry-based learning significantly stimulates students' thinking. Creative thinking often relies on more than just logic or reasoning (Koufaris, 2022). The education system requires students to think creatively, and inquiry-based learning is an approach that nurtures and hones this skill. Mathematics inquiry-based activities enhance students' interest in learning, serve as a means for acquiring mathematical knowledge, and foster a deeper understanding of the essence of mathematics. They help students develop good learning habits, view problems from multiple perspectives, and apply various strategies to solve mathematical problems (Nang et al., 2022).

### 2.2.2 Mathematical Thinking Abilities

#### *2.2.2.1 The Essence of Mathematical Thinking Ability*

Mathematical thinking ability encompasses mathematical computation skills, spatial imagination skills, and logical thinking skills, with thinking considered the core of mathematical thinking (Dewi et al., 2019). According to the "General Standard for Secondary School Mathematics Curriculum" (2017 edition, revised in 2020), students' mathematical thinking abilities include intuitive perception, observation, induction, analogy, spatial imagination, abstraction, symbolic representation, computation, data processing, deductive proof, and reflection and construction (Chen et al., 2020). Mathematical thinking ability is described from two distinct perspectives. It comprises twelve specific abilities: discovering attributes, mathematical variation, identifying similarities, mathematical reasoning, mathematical transformation, intuitive thinking, conceptual generalization, generalization of mathematical principles, adaptive generalization, discovering relationships, pattern recognition, and using cognitive blocks (Dewi et al., 2019). These abilities are categorized into conventional mathematical thinking abilities and innovative abilities. Conventional mathematical thinking includes ten aspects: numerical and geometric intuition and judgment, data collection and analysis, geometric intuition and spatial imagination, mathematical representation and modeling, and mathematical operations and transformations (Mustafa et al., 2019).

***2.2.2.2 Characteristics of Mathematical Thinking Ability***

Mathematical thinking ability shares general characteristics of thought such as abstraction, indirectness, and generalization, but it also has its unique scope (Heleni & Zulkarnain, 2018). It involves understanding, internalizing, and applying quantitative relationships, spatial forms, and mathematical structures, making it distinct from general thinking abilities (Mudrikah, 2015). The development of mathematical thinking in humans is sequential and stage-based, displaying different characteristics as age progresses. Students' mathematical thinking primarily begins with intuitive and visual thinking, often accompanied by weaker analytical and comprehension abilities, which rely mainly on intuitive appearances and experiences. By high school, students transition from intuitive to abstract logical thinking, with increased independence and thinking, often questioning and solving problems independently (Saragih & Napitupulu, 2015). This stage of development involves a shift towards dialectical abstract thinking, where students increasingly use induction and deduction to solve mathematical problems.

Therefore, secondary school students' mathematical thinking ability should exhibit the following characteristics:

(1) Purposefulness

In high school mathematics, students think toward specific goals and tasks. Teachers should design questions that are targeted to guide students' thinking and problem-solving processes (Susilawati et al., 2019). For instance, when students use vector methods to prove the cosine theorem, the key is to extract crucial information that bridges their existing knowledge and understanding. Purposeful thinking ensures that students work towards a specific direction rather than engaging in aimless or random thought processes.

(2) Flexibility

Secondary school students' mathematical thinking should be flexible, demonstrated by their ability to identify the conditions needed to solve a given mathematical problem and connect these with their existing mathematical knowledge. (Nurmanita et al., 2019) They should approach problems from multiple perspectives, exploring various problem-solving strategies, which results in solutions to problems. Flexible thinkers can simplify complex problems by identifying exceptional values of equations, showcasing their ability to adapt their thinking.

(3) Criticality

Secondary school students' mathematical thinking abilities should be critical, meaning they should question and reflect on their problem-solving processes, correct errors, and seek correct solutions (Sholehawati & Wahyudin, 2019). Students are encouraged to critically evaluate given answers and revisit their problem-solving processes when discrepancies arise, fostering an independent and critical mindset. This critical approach yields unique insights and fosters a preference for independent thinking, questioning, and problem-solving.

(4) Depth

Mathematical thinking abilities should be profound, involving in-depth contemplation of mathematical problems, a focus on underlying issues, and the avoidance of distractions from irrelevant information (Sholehawati & Wahyudin, 2019).

Secondary school students should be adept at understanding the essence of mathematical knowledge, especially when learning closely related mathematical concepts.

***2.2.2.3 Types of Mathematical Thinking Ability***

Mathematical thinking abilities can be categorized into three types based on different levels of complexity: mathematical representational thinking, mathematical logical thinking, and mathematical intuitive thinking (Heleni & Zulkarnain, 2018). Each type represents an increasing level of complexity in mathematical thought.

(1) Mathematical Representational Thinking

Representational thinking involves using objective imagery as its focus and association as its cognitive tool, primarily guiding the creation of "materialized images." Mathematical representational thinking is characterized by imagery, abstraction, and imagination. It reflects essential attributes through surface phenomena, with other mathematical thinking abilities developing from this foundation.

(2) Mathematical Logical Thinking

This type of thinking utilizes concepts that reflect the mathematical essence of objective phenomena as its cognitive material. It is based on mathematical concepts and employs logical principles to reason, define concepts, and form theorems and principles (Mudrikah, 2015). Logical thinking plays a central role in mathematical thinking abilities and is divided into formal, mathematical, and dialectical types.

(3) Mathematical Intuitive Thinking

Mathematical intuitive thinking is a direct recognition or hypothesis about the objective world and its relationships. It includes intuitive judgment, intuitive imagination, and intuitive insight (Dewi et al., 2019). This type of thinking represents a qualitative leap in cognitive ability, reflecting an insight into the essential properties of problems and representing the complexity of mathematical thinking abilities.

# CHAPTER III

# RESEARCH METHODOLOGY

## 3.1 Experimental Design

To investigate the impact of inquiry-based learning on secondary school students' mathematical thinking abilities, this study designed a survey questionnaire, a learning plan, and a mathematical thinking test. The research aims to explore the effectiveness of this learning approach in enhancing mathematical representational thinking, logical thinking, and intuitive thinking. The study targeted secondary school students, randomly selecting classes and dividing them into an experimental group and a control group. The experimental group received mathematics instruction centered on inquiry-based learning, while the control group continued with traditional direct instruction. Inquiry-based learning involved several key steps: posing questions related to the topic, encouraging students to engage in independent exploration, analyzing and explaining answers to questions, guiding students to summarize, reflect, and evaluate the exploratory learning process, and sharing and communicating the findings. Each step is designed to guide students through active participation, collaboration, and critical thinking, thereby deepening their understanding of mathematical concepts. At the beginning of the study, all participants underwent a series of mathematical tests to assess their initial levels of mathematical representational thinking, logical thinking, and intuitive thinking.

The experimental group received ongoing training in inquiry-based learning, with teachers encouraging students to ask questions, conduct independent investigations, and engage in group discussions and analysis to conclude. This process also included student reflection and evaluation of their learning experiences, as well as sharing their findings in class to strengthen their grasp and application of mathematical concepts. At the end of the study, all participants took the same mathematical tests to evaluate changes in learning outcomes. By comparing the progress in mathematical thinking abilities between the experimental and control groups, the study analyzed the effectiveness of inquiry-based learning in improving mathematical representational, logical, and intuitive thinking.

The independent variable in the study is inquiry-based learning, which encompasses several key steps: posing questions related to the topic, encouraging students to engage in independent exploration, analyzing and explaining answers to questions, guiding students to summarize, reflect, and evaluate the exploratory learning process, and sharing and communicating findings. The dependent variable is mathematical thinking ability. Based on these variables, the study designed a survey questionnaire, a learning plan, and a mathematical thinking test.

The questionnaire was divided into two sections. The first section gathered basic demographic information about the sample, including gender, age, and other demographic characteristics. The second section consisted of measurement items based on the components of inquiry-based learning. It measured students' satisfaction with various aspects of inquiry-based learning, with five items designed for each process, totaling 30 items. The questionnaire utilized a five-point Likert scale for responses. The learning plan was designed according to the steps of inquiry-based learning. The plan included specific content for developing students' mathematical thinking, teaching practices, and implementation steps. The mathematical thinking test was divided into two parts. The first part assessed students' mathematical thinking abilities before they underwent inquiry-based learning to determine any pre-existing differences. The second part evaluated students' mathematical thinking abilities after they had experienced both inquiry-based and direct instruction, measuring the differences in their mathematical thinking abilities.

## 3.2 Population and Sample Size

### 3.2.1 Population

The population consisted of Grade 3 students, including 5 classes with a total of 200 students, at Guang Ming Middle School. The subjects of this study were students from Class 1 and Class 2 in Grade 3 at Guang Ming Middle School. The Grade 3 students at this school were taught mathematics using two different methods: inquiry-based learning and direct instruction. Class 1 comprised 43 students, including 22 girls and 21 boys. Class 2 also had 43 students, with 21 girls and 22 boys. This study selected 86 students from Class One and Class Two as the research sample to investigate the impact of these two teaching methods on students' mathematics learning outcomes. The students in the third year were aged between 14 and 16 years old. By conducting an in-depth study of this group, the researcher aimed to gain a deeper understanding to support the research questions.

### 3.2.2 Samples

For this study, the subjects were the two Grade 3 classes at Guang Ming Middle School. Thus, the two existing classes served as the experimental and control groups. To ensure the reliability and scientific validity of the results, Class 1 was designated as the experimental group, which received inquiry-based learning. In contrast, Class 2 acted as the control group, continuing with direct instruction. Class 1 comprised 43 students, including 22 girls and 21 boys. Class 2 also had 43 students, with 21 girls and 22 boys. This study selected 86 students from Class One and Class Two as the research sample to investigate the impact of these two teaching methods on students' mathematics learning outcomes. This design minimized the impact of individual differences between classes on the research outcomes. The comparable number of students and the similar gender ratios in both classes helped control for potential biases related to gender and class size, making the results more generalizable and persuasive. During data analysis, the class served as the unit of analysis, and other variables (such as gender, age, and initial mathematical proficiency) were controlled to ensure that the conclusions accurately reflect the impact of inquiry-based learning on mathematical thinking abilities.

**3.2.3 Sampling Methods**

The sampling in this study was as follows: First, Class One and Class Two adopted different mathematics teaching methods (inquiry-based learning and direct instruction, respectively), which were highly relevant to the research topic. Other classes may employ varying degrees of blended learning, thereby meeting the comparative requirements of the study. Secondly, Class One and Class Two had similar student populations with a relatively balanced gender ratio and comparable overall academic performance, ensuring fairness at the starting point between the experimental and control groups. Thirdly, both classes were taught by the same group of teachers in the same academic year, with identical curriculum content and teaching resources, differing only in teaching methods. This consistency eliminated other interfering factors besides teaching methods. By comparing the two teaching methods of Class One (inquiry-based learning) and Class Two (direct instruction), this study analyzed their different impacts on students' mathematics learning outcomes, providing data support and theoretical foundations for educational practice. In this study, the two predetermined classes were used as the samples. Given the similarity in student numbers and gender ratios, the entire class was included in the research. This study employed the cluster sampling method, a common approach in educational research, particularly when evaluating the effectiveness of various teaching methods within schools or classrooms. Since the study subjects were naturally divided into two classes, these classes represented two distinct "clusters," and each received a different teaching method. Therefore, the entire class served as the sampling unit for the study. Class 1 was the experimental group (inquiry-based learning), and Class 2 was the control group (direct instruction). All students in each class participated in the study, eliminating the need for further sub-sampling. This approach simplified the research process and reduced random errors due to individual differences, as all students were exposed to their respective teaching methods and assessed for effectiveness. Through this cluster sampling method, the study directly compared the differences in mathematical thinking abilities between the two classes, effectively evaluating the impact of inquiry-based learning.

## 3.3 Data Collection

**Step 1: Study Related Theories of Inquiry-Based Learning**

This study involved an in-depth exploration of the theoretical foundations of Inquiry-Based Learning (IBL). It included a comprehensive review of existing literature to understand the concepts, principles, implementation strategies, and applications of IBL in mathematics education. Through literature analysis and theoretical study, the research aimed to provide a solid academic foundation for the design and implementation of subsequent research, ensuring that the study is grounded in robust theoretical support.

**Step 2: Develop a Satisfaction Survey**

In this step, a satisfaction survey focused on Inquiry-Based Learning was developed to gather information on students' current state of mathematical learning and thinking, as well as their attitudes toward IBL. The survey was designed based on relevant theories and research objectives, covering aspects of the IBL process and mathematical thinking abilities.

**Step 3: Pre-Test**

The pre-test was conducted to assess students' mathematical thinking levels prior to the commencement of the research. The test assessed core mathematical thinking abilities, including logical thinking, problem-solving skills, and abstract reasoning. Understanding students' cognitive abilities before exposure to different teaching methods provided a baseline for subsequent comparative analysis.

**Step 4: Inquiry-Based Learning and Direct Instruction**

In this phase, students were divided into two groups. One group received mathematics instruction through Inquiry-Based Learning (IBL), while the other group followed the direct instruction method. Students in the IBL group engaged in learning through teacher-guided questioning and self-directed inquiry. In contrast, the direct instruction group learned mathematical concepts through teacher-led lectures, demonstrations, and practice.

**Step 5: Post-Test**

After completing the instructional experiment, a post-test was conducted to reassess students' mathematical thinking abilities. The post-test used the same or similar assessments as the pre-test to determine whether significant changes in cognitive abilities had occurred. By comparing pre-test and post-test results, the study analyzed the different impacts of IBL and direct instruction on students' development in mathematical thinking.

**Step 6: Conduct the Satisfaction Survey**

Following the experiment, the satisfaction survey was administered again to collect data on students' learning experiences, feedback on teaching methods, and attitudes toward mathematics learning. The survey aimed to gather students' subjective perceptions of IBL and direct instruction, as well as their engagement and satisfaction during the experiment. The results provided additional qualitative data to help fully understand the impact of different teaching methods.

**Step 7: Analyze Data**

The final step involved a comprehensive analysis of the collected pre-test, post-test, and survey data. Statistical software was used to process the data, comparing the effects of different teaching methods on the development of mathematical thinking abilities.

## 3.4 Research Instrument

**Part 1 for Answering Research Question 1**

**Mathematical Thinking Ability Test**

The mathematical thinking ability test was designed to comprehensively evaluate students' abilities in mathematical representation thinking, logical thinking, and intuitive thinking. The test included questions targeting each of these abilities. The test items were divided into pre-test and post-test sections, administered before and after the students underwent inquiry-based learning. The purpose of these tests was to illustrate the differences in mathematical thinking abilities between inquiry-based learning and direct instruction.

To comprehensively enhance students' abilities in mathematical representation thinking, logical thinking, and intuitive mathematical thinking, the learning plans were designed around the six steps of inquiry-based learning. Each type of thinking had its learning plan, comprising the following six steps: questioning, investigation and research, analysis and interpretation, conclusion formation, reflection and evaluation, and sharing and communication.

**Learning Plans**

**Learning Plan 1: Mathematical Representation Thinking**

Duration: 180 minutes

Objective: Develop students' ability to pose questions through visual shapes and models.

(1) Pose Questions Related to the Topic

The teacher presented various geometric shapes, including polygons, circles, and solids. After observing, students posed questions related to these shapes, such as "How do you calculate the area of this shape?" or "What are the relationships between these shapes?"

(2) Encourage Student Independent Exploration

Students used tools (e.g., rulers, compasses, measuring instruments) to measure and draw shapes or use geometric software to simulate shapes and record data.

(3) Analyze and Explain Answers to Questions

Students organized measurement data, performed calculations using geometric formulas, and used charts to display the results of their analysis. Groups discussed the characteristics of each shape and its relationships with other shapes.

(4) Guide Students to Summarize

Students collaborated in groups to form conclusions about the properties of shapes, such as "How is the area of a circle calculated?" and verify these conclusions through geometric reasoning.

(5) Reflection and Evaluation of the Exploratory Learning Process

Students reviewed the entire inquiry process, identified difficulties and successes in geometric measurement and reasoning, and self-assessed through surveys or discussions.

(6) Sharing and Communication of Findings

Students presented their geometric research findings through posters or PowerPoint presentations, reported to the class, and provided feedback during discussions.

**Learning Plan 2: Mathematical Logical Thinking**

Duration: 180 minutes

Objectives: Guided students to pose key questions in logical reasoning through mathematical theorems and problems, and analyzed different proof methods and their logical foundations.

(1) Pose Questions Related to the Topic

The teacher presented classic mathematical problems, such as "Why does the Pythagorean theorem hold?" Students attempted to pose related reasoning questions, such as "Can this theorem be proven using different methods?"

(2) Encourage Student Independent Exploration

Students researched different proof methods for the Pythagorean theorem by consulting literature, using geometric tools, or employing computational software and exploring its applications in various problems.

(3) Analyze and Explain Answers to Questions

Students discussed and compared different proof methods in groups, analyzed the logical steps of the methods, and explained their validity.

(4) Guide Students to Summarize

Students selected the most effective proof method through discussion and logical reasoning and summarized the key logical steps in the reasoning process.

(5) Reflection and Evaluation of the Exploratory Learning Process

Students reviewed the different proof methods used, evaluated their logical consistency and difficulty, and discussed improvements in the reasoning process.

(6) Sharing and Communication of Findings

Students presented their proof processes using PowerPoint or whiteboards, discussed the advantages and disadvantages of different methods with the class, and shared their reasoning experiences.

**Learning Plan 3: Mathematical Intuitive Thinking**

Duration: 180 minutes

Objectives: Guide students to propose preliminary problem-solving ideas through quick judgment of the problem essence and analyze the accuracy of intuitive judgments and sources of error.

(1) Pose Questions Related to the Topic

The teacher posed open-ended questions, such as, "Given a complex function, how can you quickly estimate its limit?" Students attempted to make intuitive judgments about solving the problem.

(2) Encourage Student Independent Exploration

Students performed quick calculations and estimations in various mathematical contexts, recorded the differences between their intuitive judgments and actual results, and analyzed the reasons behind these discrepancies.

(3) Analyze and Explain Answers to Questions

Students compared their intuitive judgments with actual solutions, analyzed the errors, and identified the key factors that influenced their judgments.

(4) Guide Students to Summarize

Students inductively summarized effective intuitive judgment strategies and tested these strategies in new problem contexts.

(5) Reflection and Evaluation of the Exploratory Learning Process

Students reviewed their intuitive judgments and actual results, evaluated the effectiveness of intuitive thinking, and discussed directions for improvement.

(6) Sharing and Communication of Findings

Students shared their experiences with intuitive judgment through group discussions and class presentations, and discussed how to apply intuitive thinking to future mathematical problems more effectively.

**Part 2: Answering Research Question 2**

**Questionnaire**

The questionnaire was divided into two main sections to thoroughly understand students' basic information and their experiences and satisfaction with inquiry-based learning. The questionnaire aimed to gather effective feedback on inquiry-based learning and assess its impact on students' learning outcomes. Section 1: Basic Information Survey. This section collected students' basic demographic characteristics to provide contextual background for subsequent data analysis. Section 2: Inquiry-Based Learning Measurement Items. This part of the questionnaire was designed to assess students' overall satisfaction with inquiry-based learning and to measure their experiences with each of the six key steps involved in the process. To achieve this, the study has developed 30 measurement items covering the six crucial aspects of inquiry-based learning, with 5 items. These items were evaluated using a Likert five-point scale, ranging from "Strongly Disagree" to "Strongly Agree," allowing students to express the extent of their agreement with each statement.

5 means "Very Agree"

4 means "Agree."

3 means "Moderate."

2 means "Disagrees."

1 means "Very Disagree"

Quantitative data are analyzed by means $\bar{x}$ and S.D., and the mean value of the suitability score of expert opinions is calculated and compared with the following criteria:

A mean score of 4.51-5.00 means “very agree.” （interpreted as very high）

A mean score of 3.51-4.50 means "agree"（interpreted as high）

A mean score of 3.01-3.50 means "moderate"（interpreted as moderate）

A mean score of 1.51-3.00 means "disagree（interpreted as low）

A Mean score of 1.00- 1.50 means "very disagree"（interpreted as very low）

## 3.5 Content Validity and Reliability

### 3.5.1 Mathematical Thinking Ability Test

Validity refers to the extent to which a measurement tool or instrument accurately measures the intended content. To ensure the validity of the "Mathematical Thinking Ability Test," an evaluation was conducted. Three mathematics education experts rated the validity of the test items for this study.

Ratings were as follows:

- ✓ A rating of +1 indicates that the statement is "consistent with the definition."
- ✓ A rating of 0 indicates "uncertain whether it aligns with the definition."
- ✓ A rating of -1 indicates that the information is "not consistent with the definition."

The Index of Objective Consistency (IOC) was calculated. A content consistency index of 0.5 or greater is deemed suitable for research. The IOC analysis result was 1.00.

**3.5.2 Questionnaire**

The validity of the questionnaire was assessed using two methods: The Index of Objective Consistency (IOC) and the Kaiser-Meyer-Olkin (KMO) Measure of Sampling Adequacy.

(1) IOC (Index of Objective Consistency)

The validity analysis of the questionnaire requires three mathematics education experts to rate the questionnaire items.

Ratings are as follows:

- ✓ A rating of +1 indicates that the statement is "consistent with the definition."
- ✓ A rating of 0 indicates "uncertain whether it aligns with the definition."
- ✓ A rating of -1 indicates that the information is "not consistent with the definition."

The IOC is calculated, with a content consistency index of 0.5 or greater being suitable for research. The IOC analysis result was 1.00.

(2) KMO (Kaiser-Meyer-Olkin)

The KMO Measure of Sampling Adequacy and Bartlett's Test of Sphericity were used to determine whether the data are suitable for factor analysis. KMO values are interpreted as follows:

- ✓ Above 0.90: Indicates the scale is very suitable for factor analysis.
- ✓ Between 0.80 and 0.90: This indicates the scale is suitable for factor analysis.
- ✓ Between 0.70 and 0.80 indicates that the scale can barely support factor analysis.
- ✓ Between 0.60 and 0.70 indicates that the scale can barely support factor

analysis.

- ✓ Between 0.50 and 0.60: This indicates the scale is not recommended for factor analysis.
- ✓ Below 0.50: Indicates the scale is very unsuitable for factor analysis.

Factor analysis confirmed the structural validity of the questionnaire.

The validity of the survey questionnaire is indicated by the Kaiser-Meyer-Olkin (KMO) value. A KMO value greater than 0.7 suggests that the survey data are suitable for factor analysis. The survey data indicate that the overall KMO value is 0.856, with a significance level of 0.000, which is less than 0.05 and thus meets the significance threshold, suggesting that factor analysis can be conducted. According to the factor analysis results of each variable, the cumulative explanatory variances of Questioning, Investigation and Research, Analysis and Interpretation, Conclusion Formation, Reflection and Evaluation, and Sharing and Communication are all above 0.5, indicating their suitability for factor analysis. Through factor analysis, five valid factors were obtained: Questioning, Investigation and Research, Analysis and Interpretation, Conclusion Formation, Reflection and Evaluation, and Sharing and Communication.

Through factor analysis, items were categorized into five dimensions. Based on the dependent variable factor analysis, five items with characteristic values greater than 1 were extracted, which were consistent with the original categorization of the items. Additionally, the factor loadings of the questionnaire measurement items were all greater than 0.5, indicating good discriminant validity among the dimensions and suggesting that each dimension had good independence from the others. This demonstrates that the overall validity of the questionnaire is good, as shown in the Appendix table.

(3) Reliability Analysis of the Questionnaire

Reliability analysis is a statistical process that reflects the true extent of the measured characteristic based on the consistency or stability of the test scale results.

The more uniform the test results are, the more representative the data is of the overall situation, and the higher the reliability is. Through reliability analysis, we can understand whether the questionnaire design is reasonable and make corrections to avoid misclassification issues. Cronbach's alpha is used to evaluate the degree of internal consistency among test items. The higher the value of Cronbach's alpha, the higher the degree of consistency among items. When the reliability coefficient of a sub-scale is above 0.7, the reliability of the scale or questionnaire is considered good; when it is between 0.6 and 0.7, it is also acceptable; and when the reliability coefficient of the overall scale needs to reach 0.8 or higher, it indicates good overall reliability. Furthermore, the corrected item-total correlation for all items is at least 0.4, and the reliability decreases to varying degrees when any item is deleted.

The reliability of the survey data was assessed using Cronbach's Alpha indices. The study found that Cronbach's Alpha for Questioning was 0.902, for Investigation and Research was 0.892, for Analysis and Interpretation was 0.869, for Conclusion Formation was 0.841, for Reflection and Evaluation was 0.887, and for Sharing and Communication was 0.908, as shown in Table 4-2. The research data indicate that all Cronbach's Alpha values exceed 0.8, suggesting good reliability of the survey questionnaire, as shown in the Appendix table.

## 3.6 Data Analysis

### 3.6.1 Descriptive Statistics

In this study, frequency, mean, and standard deviation were used for a comprehensive preliminary analysis of the collected data to understand the basic characteristics and provide a foundation for subsequent inferential statistics. Descriptive statistics primarily included statistical analyses of questionnaire data and mathematics test scores to offer insights into student performance and satisfaction.

For the questionnaire data, the focus was on summarizing and generalizing responses to each measurement item using frequency, mean, and standard deviation. This study calculated the frequency distribution for each item and determined the proportion of responses for each option. By computing measures such as the mean and standard deviation, the study assessed overall student satisfaction and feedback on various aspects of inquiry-based learning. The mean provided an overview of students' general attitudes toward each measurement area. At the same time, the standard deviation revealed the degree of dispersion in responses, indicating the level of consensus or disagreement among students. The study calculated the total scores and means for different aspects (e.g., questioning, research, etc.) to evaluate the impact of each step on the student's learning experience. These statistical results helped identify which aspects of inquiry-based learning have a significant impact on students and which areas might need improvement.

For the mathematics test scores, descriptive statistics helped assess students' mathematical thinking abilities at various stages of the testing process. The study calculated the mean scores for each test item in the pre-test and post-test to evaluate student performance across different testing phases. By calculating the average scores, standard deviations, and score distributions for each test, the study identified overall performance trends. It recognized strengths and weaknesses in students' mathematical representational thinking, logical thinking, and intuitive thinking. Additionally, comparisons were made between pre-test and post-test scores to analyze students' abilities before and after the inquiry-based learning intervention. This comparison was achieved by calculating the mean difference and rate of change between pre-test and post-test scores, which helped determine the extent to which inquiry-based learning impacts students' mathematical thinking abilities.

**3.6.2 Inferential Statistics**

This study employed a quantitative research method. As part of the data collection procedure, erroneous questionnaires, outliers, and missing values were removed from the dataset. After data cleaning, the data were organized using Microsoft

Excel and analyzed using the Statistical Package for the Social Sciences (SPSS). To compare the mathematics test scores of students in the experimental group and the control group before and after the inquiry-based learning intervention, an independent samples t-test was used. Paired samples t-tests are applied to compare the means of two related samples, such as pre-test and post-test scores.

## 3.7 Research Ethics

This study strictly adhered to ethical guidelines, with the institution's research committee fully involved and approving the ethical review throughout the process. Before the commencement of the study, all participants were informed about the nature, purpose, and procedures of the research, including potential risks and benefits. With informed consent, they voluntarily decided whether to participate and signed both written and verbal consent forms. To protect the rights and interests of the participants, they were informed of their right to withdraw from the study at any stage without providing a reason. Additionally, they had the right to choose not to answer questions or to request that the researchers destroy the information already provided. To ensure data privacy and anonymity, all information unrelated to personal identity is protected. Participants' responses were strictly confidential. The storage period for research data was specified as three years after the study's completion to meet the requirements of research archiving and academic review. During this period, the stored files were subject to stringent security measures and were accessible only by the research team. After three years, all data was destroyed by shredding physical documents to protect the rights of information providers. The research team adhered to the principles of trust. The study implemented informed consent, ensured confidentiality of information, and protected participant privacy, creating a trustworthy research environment.

# CHAPTER IV

# ANALYSIS RESULT

Based on the analysis of Chapters 1 to 3, this chapter collects and analyzes data to verify the hypotheses. The chapter utilizes SPSS software to validate each research hypothesis, employing descriptive statistical analysis (frequency, percentage, mean, and standard deviation) and independent-sample t-tests to analyze the sample distribution and demographic variables (gender, age, GPA) of students from Class 1 and Class 2, Grade 3 of Guang Ming Middle School, as well as their attitudes towards inquiry-based teaching as a means to enhance mathematical thinking skills. This study collects and analyzes data using questionnaires, learning plans, and test. The questionnaire covers basic student information and attitudes towards inquiry-based teaching, encompassing questioning, investigation and research, analysis and interpretation, conclusion formation, reflection and evaluation, and sharing and communication. A total of 86 questionnaires were collected.

The mathematical thinking ability test consists of pre-test and post-test items. The pre-test items include mathematical representational thinking, logical thinking, and intuitive thinking. The pre-test measured and ensured that there was no difference in the mathematical thinking of students before they received inquiry-based teaching and traditional teaching. The post-test items also cover mathematical, representational, logical, and intuitive thinking. The post-test is designed to measure any differences in the mathematical thinking levels of students after they have undergone inquiry-based teaching and traditional teaching. In this study, all 43 students from Class 1, Grade 3, participated in the experiment as the experimental group (inquiry-based teaching). In comparison, all 43 students from Class 2 and Grade 3 participated as the control group (direct learning).

## 4.1 Findings of Students' Attitudes Towards Inquiry-Based Teaching

The students in Classes 1 and 2, as well as those in Grade 3 of Guang Ming Middle School, demonstrate a relatively balanced distribution in demographic variables; however, they also exhibit some notable differences and characteristics. For gender, boys and girls account for 50% each, with 43 boys and 43 girls. This indicates a balanced gender ratio within the classes, avoiding a situation where one gender is overly dominant, and provides equal observation conditions for teaching research, especially when analyzing gender differences. There are significant differences in age distribution. Thirty-eight students (44.2%) are under 14 years old, belonging to the younger student group, while 18 students (20.9%) are 14 years old, and another 18 (20.9%) are over 15 years old, accounting for a proportion. Additionally, 12 students (14%) are 15 years old. For academic performance (GPA), most students' GPAs are concentrated in the range of 2.6-3.0, with 40 students (46.5%) falling into this relatively moderate performance bracket. There are 25 students (29.1%) with a GPA between 2.1 and 2.5; these students exhibit a moderate level of academic performance. Only a few students have a GPA higher than 3.5, with eight students (9.3%), and a similarly small number have a GPA below 2.0, totaling ten students (11.6%). There are a few students with excellent or poor performance. Moreover, scores are concentrated in the middle range.

Table 4.1 Frequency and Present Classified by Gender, Age, and GPA

| Items | Options | Frequency | Percent% |
|---|---|---|---|
| | Male | 43 | 50.0 |
| Gender | Female | 43 | 50.0 |
| | Under 14 | 38 | 44.2 |
| | 14 | 18 | 20.9 |
| | 15 | 12 | 14.0 |
| Age | Over 15 | 18 | 20.9 |
| | Under 2 | 10 | 11.6 |
| | 2.1-2.5 | 25 | 29.1 |
| | 2.6-3.0 | 40 | 46.5 |
| GPA | 3.1-3.5 | 3 | 3.5 |

| Items | Options | Frequency | Percent% |
|---|---|---|---|
| | Over3.5 | 8 | 9.3 |
| Total | | 86 | 100.0 |

Overall, the gender ratio of students is very balanced and suitable for gender-related analysis. The age distribution shows a certain degree of variation. Academic performance (GPA) exhibits a trend of concentration in the middle range, with a minority of students showing extreme academic performance (either excellent or lagging). This indicates that there may not be significant academic differentiation within the class. It is also necessary to consider providing personalized support for the few students with outstanding or poor academic performance. When conducting classroom teaching, teachers need to consider students' age and academic differences, providing extra help for those with slightly poorer academic performance and offering more cognitive guidance to younger students.

The data collected from the survey questionnaire have good reliability and validity. The research data present descriptive statistics on students' attitudes toward inquiry-based teaching, specifically categorized into six dimensions: questioning, investigation and research, analysis and interpretation, conclusion formation, reflection and evaluation, and sharing and communication. Each dimension contains multiple items, presenting the mean and standard deviation (Std. Deviation) for each item. See Table 4.2 for details.

In the questioning dimension, the mean range is from 3.36 to 3.55, with an overall mean of 3.45, and the standard deviations are between 1.07 and 1.27. The data indicate that the interpretation of the questioning dimension is categorized as "moderation." These results indicate that students' performance in the questioning stage is slightly above average; however, the distribution of scores is relatively dispersed, suggesting significant differences in students' abilities or attitudes.

In the investigation and research dimension, students have a slightly higher mean, with an overall mean of 3.60 and a mean range of 3.51 to 3.71. The standard deviations range from 0.984 to 1.306. The interpretation of the investigation and research dimension is classified as "high." These suggest that students have a higher level of engagement in investigation and research, and compared to the Questioning dimension, there is less variation among students.

The mean for the analysis and interpretation dimension is 3.50, with item means ranging from 3.30 to 3.63 and standard deviations ranging from 1.11 to 3.08. The interpretation of the analysis and interpretation dimension is categorized as "moderation." Although the mean for this dimension is slightly lower, the standard deviations indicate some variation in students' analytical and interpretive abilities. These may suggest that some students perform more prominently at this stage.

The mean for the conclusion formation dimension is 3.69, with a mean range of 3.50 to 3.81 and standard deviations between 1.03 and 1.31. The interpretation of the conclusion formation dimension is classified as "high." The higher mean indicates that students perform relatively well in the conclusion formation stage, and the standard deviations show that there is still some variability among students.

The reflection and evaluation dimension has the highest mean of 3.83, with individual item means ranging from 3.71 to 4.06 and relatively small standard deviations, ranging from a low of 0.82 to a high of 1.21. The interpretation of the reflection and evaluation dimension is categorized as "high." These indicate that students have a strong sense of reflection and evaluation of their learning process and show positive performance. The data for this dimension suggest that most students have high engagement and relatively balanced performance at this stage.

The overall mean for the sharing and communication dimension is 3.61, with a mean range of 3.43 to 3.71 and standard deviations between 1.12 and 1.25. The interpretation of the sharing and communication dimension is also classified as "high." The mean is lower, suggesting that students' performance in sharing and communication may not be as strong as in other dimensions.

Table 4.2 Items, Mean, Standard Deviation, and Interpretation of Satisfaction

| Dimension | Mean | Std. Deviation | Interpretion |
|---|---|---|---|
| Questioning | 3.45 | 1.17 | Moderation |
| Investigation and Research | 3.60 | 1.15 | High |
| Analysis and Interpretation | 3.50 | 1.20 | Moderation |
| Conclusion Formation | 3.69 | 1.14 | High |
| Reflection and Evaluation | 3.83 | 1.02 | High |
| Sharing and Communication | 3.61 | 1.18 | High |
| Total | 3.61 | 1.15 | High |

The dimensions of reflection and evaluation, conclusion formation, and investigation and research are categorized as "High," indicating strong student engagement and performance. In contrast, the questioning, analysis, and interpretation dimensions are rated as "Moderation," revealing areas for potential improvement. These findings suggest a need for pedagogical interventions to foster students' critical thinking and analytical competencies, enabling more comprehensive and effective learning outcomes. Overall, students have a generally positive attitude towards inquiry-based teaching, with the most prominent performance in the reflection and evaluation and conclusion formation dimensions, reflecting their activity and confidence in the later stages of the learning process (i.e., conclusion drawing and reflection/evaluation stages). However, the means for the questioning and communication dimensions are relatively lower, and the standard deviations are high. These suggest that teachers should pay more attention to these aspects in their teaching, potentially guiding students to ask better questions and share and communicate more effectively.

## 4.2 Investigation on the Impact of Inquiry-Based Teaching on Mathematical Thinking Ability

The subjects of this study are the students in Classes 1 and 2, as well as Grade 3 of Guang Ming Middle School. Students in Grade 3 of this school learn mathematics through two methods: inquiry-based teaching and direct learning. Class 1, Grade 3 served as the experimental group (inquiry-based teaching), while Class 2, Grade 3 served as the control group (direct learning). Tests were conducted twice, once before the start of the study (pre-test) and once after its conclusion (post-test). The pre-test aims to assess students' baseline mathematical thinking abilities before they receive different teaching methods, while the post-test assesses changes in their abilities after receiving instruction. The test content should align with the curriculum outline and cover questions related to mathematical representational thinking, logical thinking, and intuitive thinking.

The achievements of inquiry-based learning are better than direct instruction in improving secondary school students' mathematical thinking abilities.

1. Test the variance of class inquiry-based learning equal to the variance of class direct instruction (before learning)

$\sigma^2_{inquiry}$: variance of class inquiry-based learning

$\sigma^2_{direct}$: variance of class direct instruction

$H_0$: $\sigma^2_{inquiry} = \sigma^2_{direct}$

$H_1$: $\sigma^2_{inquiry} \neq \sigma^2_{direct}$

Table 4.3 Comparison of the Variance of Class Inquiry-Based and the Variance of Class Direct Instruction (Before Learning)

| **Class** | **df** | **Mean** | **Variance** | **F** |
|---|---|---|---|---|
| Inquiry-Based | 42 | 7.67 | 1.35 | 0.92 |
| Direct Instruction | 42 | 7.67 | 1.46 | |

$$F_{0.025,42,42} = 0.54$$

$$F_{0.975,42,42} = 1.85$$

$$F_{0.025,42,42} = 0.54 < F_{compute} = 0.92 < F_{0.975,42,42} = 1.85$$

Accept null hypotheses. It means the test variance of class inquiry-based learning equals the variance of class direct instruction. After learning, use the t-test with equal variance.

2. Test the mean achievement of secondary school students in inquiry-based learning to assess their mathematical thinking abilities.

$\mu_{inquiry}$: mean the achievement of inquiry-based learning

$\mu_{direct}$: mean the achievement of direct instruction

$H_0: \mu_{inquiry} = \mu_{direct}$

$H_1: \mu_{inquiry} > \mu_{direct}$

Table 4.4 Comparison of the Average Achievement of Inquiry-Based Learning and Average Achievement of Direct Instruction

| **Class** | **N** | **Mean** | **Variance** | **df** | **t** |
|---|---|---|---|---|---|
| Inquiry-Based | 43 | 13.84 | 0.95 | 84 | 6.501 |
| Direct Instruction | 43 | 11.79 | 3.31 | | |

$$t_{0.05,84} = 1.66$$

$$t_{compute} = 6.501 > t_{0.05,84} = 1.66$$

Reject null hypotheses: the achievements of inquiry-based learning equal the achievements of direct instruction. Therefore, the achievement of inquiry-based learning is superior to

# CHAPTER V

# CONCLUSION AND DISCUSSION

## 5.1 Conclusion

**Part 1 Answering Research Question 1**

Before implementing inquiry-based teaching, a data test was conducted among students in Classes 1 and 2, as well as Grade 3, at Guang Ming Middle School, yielding 86 valid responses. An independent sample t-test was performed on the students' scores in the two classes. The research results showed a t-value of 0.00, degrees of freedom (df) of 84, and a significance value (Sig. 2-tailed) of 1.00. Since the significance value was 1.00, much greater than 0.05, it indicated that there was no statistically significant difference in the test scores between the two classes. After implementing inquiry-based teaching, a follow-up data test was conducted among the students of these two classes, and a t-test was performed. Based on the results of the t-test, it could be concluded that the average scores of the experimental group (inquiry-based teaching) in the post-test were significantly higher than those of the control group (direct instruction), indicating that inquiry-based teaching had a significant positive impact on students' test scores. Therefore, the research results suggest that the hypothesis that inquiry-based learning is more effective than direct instruction in improving middle school students' mathematical thinking abilities holds.

Before the implementation of inquiry-based teaching, the test scores of Class 1 and Class 2, as well as Grade 3 at Guang Ming Middle School, showed no significant difference. This meant that, before the experiment began, the students in the two classes had a balanced mathematical foundation and ability, providing a reasonable basis for comparison in the subsequent teaching experiment. Following the implementation of inquiry-based teaching, the experimental group's mean scores were significantly higher than those of the control group in the post-test. This result demonstrated the effectiveness of inquiry-based teaching in improving students' mathematics scores and further illustrated its enhancement of students' mathematical thinking abilities. Inquiry-based learning emphasizes the process of students asking questions, investigating, analyzing, and forming conclusions independently, which aligns well with the logical reasoning, critical thinking, and abstract thinking abilities required for mathematical thinking.

In contrast, direct instruction typically focuses more on teachers' explanations and students' memorization and imitation, which can improve students' mastery of basic concepts and skills in the short term but may be limited to fostering deep mathematical thinking. In inquiry-based teaching, students actively explore problems and learn to view issues from different perspectives, thereby developing more complex ways of thinking. This process is not just about finding the correct answer but about understanding the mathematical structures behind the problems and how to apply existing knowledge to solve new problems. Inquiry-based teaching also provides students with more opportunities for reflection and evaluation, which is particularly important for mathematical thinking. Students can examine processes, identify problems, and revise their ideas. The cultivation of this ability is undoubtedly crucial for mathematical thinking. Conversely, direct instruction rarely provides students with such opportunities, and students often judge right and wrong based on the standard answers given by teachers, lacking the space for independent examination and evaluation.

Thus, by giving students more autonomy and encouraging them to continuously ask questions, reflect, and improve during the learning process, inquiry-based teaching enables students to make significant progress in mathematical thinking. Compared to direct instruction, it more effectively promotes the development of students' logical reasoning, problem-solving, and critical thinking abilities, enabling them to solve problems and understand the underlying mathematical principles. This enhancement of abilities is reflected in academic performance and has an impact on students' future learning and life.

**Part 2 for Answering Research Question 2**

The research data presents descriptive statistics on students' attitudes towards inquiry-based teaching, specifically categorized into six dimensions: posing questions, investigation and research, analysis and interpretation, forming conclusions, reflection and evaluation, and sharing and communication. Overall, students hold a positive attitude toward inquiry-based teaching, with outstanding performance in the dimensions of forming conclusions. The results indicate that students are more active and confident in the later stages of the learning process, specifically in drawing conclusions and reflecting on and evaluating their work. However, the mean values for posing questions and sharing are low, indicating significant individual differences among students.

These results suggest that inquiry-based teaching is an effective method for enhancing mathematical thinking abilities. Therefore, based on the research findings, the hypothesis that student satisfaction increases through inquiry-based learning holds. In an open learning environment, differences in students' thinking abilities and expressive skills may be magnified, with some students finding that their pace in inquiry-based teaching may require less guidance and support. Thus, although students' attitudes towards inquiry-based teaching are overall positive, this positivity is not evenly distributed. To address the challenges in posing questions and sharing/communicating, incorporating more guiding questions or adopting group cooperation may help those more passive students better integrate into the inquiry-based learning process. These results suggest that when implementing inquiry-based teaching, teachers need to flexibly adjust their teaching methods to meet the diverse needs of students, ensuring that each student gains a meaningful learning experience across all dimensions.

## 5.2 Discussion

**Part 1 Answer Research Question No. 1**

1. How does inquiry-based learning compare to direct instruction in enhancing mathematical thinking abilities among secondary school students?

Inquiry-based teaching promotes the in-depth development of students' mathematical thinking in various ways. It emphasizes students' ability to pose questions, explore different problem-solving strategies, and construct their mathematical understanding during the learning process. This viewpoint aligns with the research by Oliver et al. (2019) and Ibrahim et al. (2020). Compared to the knowledge transmission approach of direct instruction, inquiry-based teaching stimulates students' active thinking and logical reasoning abilities. Inquiry-based teaching encourages students to seek multiple solutions to gain a deeper understanding of the essence of mathematical concepts, as reflected in the post-test results, where students in the experimental group performed significantly better than those who received only direct instruction.

The reason why inquiry-based teaching can promote the in-depth development of students' mathematical thinking lies in its emphasis on students playing a more active role in the learning process. Students engage in learning by posing questions and exploring different problem-solving strategies. This method encourages students to think about problems from multiple angles, try various possible solutions, and ultimately construct their unique understanding of mathematical concepts in this process. These conclusions are consistent with the research findings of Ting et al. (2019) and Nang et al. (2022). Inquiry-based teaching provides students with an open learning environment that enables them to grow through trial and error, fostering a deeper understanding of the subject matter. Each attempt to solve a problem, even if it leads to helping students gain a deeper understanding of the principles behind mathematics, thereby promoting the development of their abstract thinking abilities. This cultivation of abstract thinking is not achieved by imparting fixed problem-solving steps but through repeated exploration and comparison of different solutions. Collaborative learning in inquiry-based teaching also enhances students' self-confidence and their ability to work in teams. When exploring complex problems, students can discuss with each other, share their ideas and thoughts, and gain new inspiration from these exchanges. This viewpoint is consistent with the research by Koufaris (2022) and Chen et al. (2020). This collision of collective wisdom enhances students' understanding of mathematical problems and helps them face unknown challenges more confidently in future learning.

Therefore, inquiry-based teaching stimulates students' curiosity, learning, and participation in the reconstruction of knowledge. This method helps students achieve progress in mathematical thinking and enhances their ability to solve practical problems. In this process, students learn how to think critically, pose questions, and collaborate, thereby laying a solid foundation for their future academic development.

**Part 2 Answer Research Question No. 2**

2. What are the students' attitudes towards inquiry-based learning as a method for enhancing mathematical thinking abilities in secondary school?

Research data presents descriptive statistics on students' attitudes toward inquiry-based teaching, categorized into six dimensions: questioning, investigation and research, analysis and interpretation, conclusion formation, reflection and evaluation, and sharing and communication. In the questioning dimension, the mean is 3.45, which means "agree" (interpreted as high). For the investigation and research dimension, the mean is 3.60, indicating "agree" (interpreted as high). The mean for the analysis and interpretation dimension is 3.50, which means "moderate" (interpreted as moderate). The mean for the conclusion formation dimension is 3.69, signifying "agree" (interpreted as high). The reflection and evaluation dimension has a mean of 3.83, indicating an "agree" response (interpreted as high). Overall, students hold a generally positive attitude towards inquiry-based teaching, with the most prominent performance observed in the reflection and evaluation and conclusion formation dimensions, reflecting students' activity and confidence in the later stages of the learning process (i.e., conclusion drawing and reflection/evaluation stages). However, the means for the questioning, sharing, and communication dimensions are relatively lower, with larger standard deviations, indicating significant individual differences among students in these areas.

According to the research findings, students' attitudes toward inquiry-based teaching exhibit a positive trend, particularly in the crucial stages of reflection, evaluation, and conclusion formation. Students demonstrate considerable confidence in the later stages of the learning process when they continuously reflect on and evaluate the content they have learned. This confidence is their ability to draw clear mathematical conclusions and summarize them. At these stages, students' average attitude scores are higher, indicating that they have accepted the challenges posed by inquiry-based teaching and received positive feedback, thereby gradually establishing a sense of control and satisfaction in their learning. This later-stage autonomy enhances their mathematical thinking abilities and deepens their consideration of complex problems. This viewpoint aligns with the research by Sholehawati (2019).

Students' performance in questioning, sharing, and communication is relatively less prominent. The lower score in the questioning dimension suggests that students may lack sufficient confidence to actively ask questions or express their doubts when faced with unfamiliar or complex mathematical problems. Such situations may stem from various reasons, such as students' insufficient understanding of mathematical problems or unfamiliarity with this open-ended way of thinking in the classroom. In an inquiry-oriented classroom, asking questions is the first step in the learning process, and students' differences in this aspect suggest that some may face challenges in adapting to this teaching method. This viewpoint is consistent with the research by Nurmanita et al. (2019).

The sharing and communication aspect is noteworthy. Inquiry-based teaching emphasizes cooperative learning and group discussions; however, the data indicate that students' enthusiasm in this area is relatively low, with significant variation among them. This may reflect students' varying levels of comfort with classroom interaction, with some students being more adept at independent thinking and others feeling less comfortable when sharing their thoughts. This phenomenon suggests that when promoting inquiry-based teaching, teachers may need to consider encouraging students to be more active in communication during cooperative learning and establish a more inclusive classroom atmosphere so that all students can feel safe and supported in the discussion environment. This viewpoint aligns with the research by Susilawati et al. (2019).

The results demonstrate students' positive attitudes towards inquiry-based teaching, especially their outstanding performance in the later reflection and summary stages. This suggests that inquiry-based teaching enables students to gain a deeper understanding of mathematical problems and enhances their thinking flexibility and self-evaluation abilities. In this teaching mode, students must reflect on their learning paths, evaluate their thinking processes, and adjust their approach based on these reflections, thereby laying a solid foundation for their mathematical learning.

## 5.3 Implications for Practice and Research Future

Inquiry-based teaching offers students opportunities for autonomous exploration. During this process, students learn to discover the essence of problems through questioning and investigation and attempt to solve them using various strategies. It helps students gain a deeper understanding of mathematical concepts. Inquiry-based teaching also exercises students' ability to cope with complex problems. Such practice is highly conducive to cultivating students' patience and strategic awareness in problem-solving, enabling them to no longer settle for a single solution but to consider the possibilities of problems from different angles. Inquiry-based teaching emphasizes cooperative learning and communication among students. In practical applications, through group discussions and the sharing of ideas, students can gain new perspectives and reflect deeply on their thinking processes through interactions with peers. This exchange of collective wisdom enables students to recognize diversified solution paths, thereby enriching their levels of thinking. Although research indicates that individual differences in sharing and communication persist, teachers can gradually cultivate students' confidence and abilities in these areas by facilitating more active discussions and cooperation.

A notable practical effect of inquiry-based teaching is the enhancement of students' reflective abilities. In the later stages of learning, students continuously reflect on and evaluate their problem-solving skills and then test and revise them, which promotes flexibility in their thinking. Reflection not only enables students to become aware of their learning deficiencies but also helps them make targeted improvements in subsequent learning. Through continuous adjustment and optimization, students' mathematical thinking is further enhanced, and their understanding of knowledge becomes more comprehensive. In practical applications, these advantages of inquiry-based teaching enable students to transcend their understanding of specific mathematical problems and cultivate broader mathematical literacy. It provides students with a space for continuous growth, enabling them to cope with current learning challenges and lay a solid foundation for future mathematical studies. The success of this teaching method relies on guidance from teachers, an open classroom atmosphere, and active student interaction. Through these practical aspects, inquiry-based teaching enables students to become more autonomous and confident in their mathematics learning, providing long-term support for their intellectual development.

## 5.4. Recommendations for Future Research

Future research should further explore the impact of inquiry-based learning and direct instruction on middle school students' mathematical thinking abilities, examining its mechanisms and potential benefits from multiple perspectives. Studies can analyze the applicability of these teaching methods in other subjects. Individual differences among students also merit attention, as varying learning styles, cognitive levels, motivations, and gender or personality traits may moderate the effectiveness of inquiry-based learning, suggesting potential directions for future personalized instruction. Research on long-term effects is also crucial. Through longitudinal designs, we can ascertain whether the impact of inquiry-based learning on students' mathematical thinking abilities persists and whether it exerts a profound influence on other dimensions, such as creativity or complex problem-solving skills. Comparing different forms of inquiry-based learning to explore their specific differences and optimal applications in diverse teaching scenarios. Combining inquiry-based learning with modern information technology represents a key focus for future research. Digital resources and online learning platforms can provide technical support for inquiry-based learning. Studying how to utilize these tools effectively may further optimize students' learning experiences and outcomes. Integrating teachers' professional competencies with inquiry-based teaching methods warrants in-depth exploration, particularly regarding how training can assist teachers in better designing questions, managing classrooms, and guiding student discussions and collaborations. Cross-cultural research is another perspective. The acceptability and effectiveness of inquiry-based learning may vary across different cultural contexts, with educational practices in rural versus urban areas, regions, or even countries offering unique cases. In addition, analyzing the relationship between students' attitudes towards inquiry-based learning and their actual learning outcomes, particularly examining whether cognitive load moderates or mediates this relationship, is worthwhile. Future studies should design diversified experiments, such as expanding sample sizes or introducing hybrid teaching groups. Through research in these directions, we can deepen our understanding of inquiry-based learning and provide more practical recommendations for optimizing teaching practices.

## 5.5. Limitations of Study

Although this study has made some significant findings regarding the impact of inquiry-based learning and direct instruction on enhancing middle school students' mathematical thinking abilities, it still has the following limitations: The sample size is small and concentrated in a single class, with only 43 students from Class 1, Grade 3, serving as the experimental group, which limits the generalizability and representativeness of the research results. The study duration is relatively short, assessing teaching effectiveness solely through pre-test and post-test comparisons, and cannot capture the long-term impact of inquiry-based learning. The enhancement of mathematical thinking abilities may require a period of cultivation. During the implementation of inquiry-based learning, teachers' instructional proficiency, classroom management skills, and student participation significantly influenced the research outcomes; however, this study did not provide a detailed analysis of these variables. These results cannot distinguish the effects of the inquiry-based learning method from those of external factors. This study primarily employed quantitative research methods and did not further explore students' authentic learning experiences and teachers' instructional strategies through qualitative methods such as interviews or classroom observations. This single data source may have omitted some deeper-level information. There are also certain limitations in the design of the research tools. Limitations in the testing tool may affect the accuracy of the research conclusions. The limitations of this study primarily stem from the constraints of sample size and scope, the study's short duration, methodological homogeneity, and inadequate control of influencing factors. These shortcomings provide directions for improvement in future research, such as expanding the sample scope, combining quantitative and qualitative research methods, extending the research period, and deeply exploring the impact of individual differences among students and external factors on teaching effectiveness.

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# APPENDICES

## Questionnaire

Dear Students,

Thank you for your participation in this questionnaire survey. The survey will be conducted anonymously, and your relevant information will be kept confidential. Thank you again for your cooperation.

**Part I:**

1. Gender? A Male B Female

2. Age:

A Under 14

B 14

C 15

D Over 15

3. How about your GPA

A Under 2

B 2.1-2.5

C 2.6-3.0

D 3.1-3.5

E Over3.5

**Part II:**

Please judge to what extent you agree with the following statement; please choose the most appropriate option and mark the corresponding number "√". The questionnaire used a Likert scale, ranging from 1 to 5, in which 1 indicates strongly disagree (or strongly disagree), 2 indicates disagree (or disagree), 3 indicates neutral, 4 indicates agree (or agree), and 5 indicates strongly agree (or strongly agree)

| Measurement Item | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Do you think that asking questions during the inquiry process is helpful for learning? | | | | | |
| Were you satisfied that you had enough opportunities to ask questions? | | | | | |
| Do you feel that your teacher's feedback on the questions you asked was satisfactory? | | | | | |
| Did you feel clear in your thinking during the questioning process? | | | | | |
| Were you able to effectively translate questions into learning objectives? | | | | | |
| Were you satisfied that the research study helped you understand the learning content more deeply? | | | | | |
| Were you satisfied with the resources provided by the research study? | | | | | |
| Were you satisfied with the tasks during the research study? | | | | | |
| Were you able to effectively organize and document the results of the research study? | | | | | |
| Were you satisfied with the collaboration that took place during the research study? | | | | | |
| Were you satisfied with the clarity of the data analysis process? | | | | | |
| Were you able to effectively extract useful information from the data? | | | | | |
| Were you able to relate the analysis results to the study's content? | | | | | |
| Were you satisfied with the task of analyzing and interpreting the data? | | | | | |
| Were you satisfied with the support your teacher provided during the analysis and interpretation stage? | | | | | |

| Measurement Item | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Were you able to draw clear conclusions based on the analysis results? | | | | | |
| Were you satisfied with the process of forming conclusions? | | | | | |
| Were you able to relate the conclusions effectively to the original question? | | | | | |
| Were you satisfied with the conclusions formed? | | | | | |
| Are you satisfied with the resolution of difficulties encountered during the conclusion formation stage? | | | | | |
| Do you engage in regular self-reflection to assess the effectiveness of your learning? | | | | | |
| Were you able to identify the challenges you encountered in the learning process? | | | | | |
| Are you satisfied with the contribution of the reflection and assessment process to improving your learning? | | | | | |
| Are you satisfied with the feedback on reflection and assessment provided by your teachers? | | | | | |
| Are you satisfied with the significance of the reflection and assessment process? | | | | | |
| Are you satisfied with how the process of sharing your learning has helped your learning? | | | | | |
| Were you able to articulate your learning outcomes and conclusions clearly and concisely? | | | | | |
| Do you feel that communication with your classmates helped you to understand your learning? | | | | | |
| Were you satisfied with the opportunities for sharing and communication? | | | | | |
| Are you satisfied that you gained new insights from your classmates' sharing? | | | | | |

# Lesson Plan

**Learning Plan 1: Mathematical Representation Thinking**

Duration: 180 minutes

Objective: Develop students' ability to pose questions through visual shapes and models.

(1) Pose Questions Related to the Topic

The teacher will present various geometric shapes, including polygons, circles, and solids. After observing, students will pose questions related to these shapes, such as "How do you calculate the area of this shape?" or "What are the relationships between these shapes?"

(2) Encourage Student Independent Exploration

Students will use tools (e.g., rulers, compasses, measuring instruments) to measure and draw shapes or use geometric software to simulate shapes and record data.

(3) Analyze and Explain Answers to Questions

Students will organize measurement data, perform calculations using geometric formulas, and use charts to display the results of their analysis. Groups will discuss the characteristics of each shape and its relationships with other shapes.

(4) Guide Students to Summarize

Students will collaborate in groups to form conclusions about the properties of shapes, such as "How is the area of a circle calculated?" and verify these conclusions through geometric reasoning.

(5) Reflection and Evaluation of the Exploratory Learning Process

Students will review the entire inquiry process, identify difficulties and successes in geometric measurement and reasoning, and self-assess through surveys or discussions.

(6) Sharing and Communication of Findings

Students will present their geometric research findings through posters or PowerPoint presentations, report to the class, and provide feedback during discussions.

**Learning Plan 2: Mathematical Logical Thinking**

Duration: 180 minutes

Objectives: Guide students to pose key questions in logical reasoning through mathematical theorems and problems, and analyze different proof methods and their logical foundations.

(1) Pose Questions Related to the Topic

The teacher will present classic mathematical problems, such as "Why does the Pythagorean theorem hold?" Students will attempt to pose related reasoning questions, such as "Can this theorem be proven using different methods?"

(2) Encourage Student Independent Exploration

Students will research different proof methods for the Pythagorean theorem by consulting literature, using geometric tools, or employing computational software, and exploring its applications in various problems.

(3) Analyze and Explain Answers to Questions

Students will discuss and compare different proof methods in groups, analyze the logical steps of the method, and explain their validity.

(4) Guide Students to Summarize

Students will select the most effective proof method through discussion and logical reasoning and summarize the key logical steps in the reasoning process.

(5) Reflection and Evaluation of the Exploratory Learning Process

Students will review the different proof methods used, evaluate their logical consistency and difficulty, and discuss improvements in the reasoning process.

(6) Sharing and Communication of Findings

Students will present their proof processes using PowerPoint or whiteboards, discuss the advantages and disadvantages of different methods with the class, and share reasoning experiences.

**Learning Plan 3: Mathematical Intuitive Thinking**

Duration: 180 minutes

Objectives: Guide students to propose preliminary problem-solving ideas through quick judgment of the problem essence and analyze the accuracy of intuitive judgments and sources of error.

(1) Pose Questions Related to the Topic

The teacher will pose open-ended questions, such as, "Given a complex function, how can you quickly estimate its limit?" Students will attempt to make intuitive judgments about solving the problem.

(2) Encourage Student Independent Exploration

Students will perform quick calculations and estimations in various mathematical contexts, record the differences between their intuitive judgments and actual results, and analyze the reasons behind these discrepancies.

(3) Analyze and Explain Answers to Questions

Students will compare their intuitive judgments with actual solutions, analyze the errors, and identify the key factors that influence their judgments.

(4) Guide Students to Summarize

Students will inductively summarize effective intuitive judgment strategies and test these strategies in new problem contexts.

(5) Reflection and Evaluation of the Exploratory Learning Process

Students will review their intuitive judgments and actual results, evaluate the effectiveness of intuitive thinking, and discuss directions for improvement.

(6) Sharing and Communication of Findings

Students will share their experiences with intuitive judgment through group discussions or class presentations, and discuss how to better apply intuitive thinking to

# Test

**Mathematical Thinking Abilities Test Items**

**Pre-Test Items**

**1. Mathematical Representational Thinking**

(1) Which of the following figures has all sides equal?
A. rectangle
B. trapezoid
C. hexagon
D. square

(2) What is the perimeter of a square triangle whose sides are 5 centimeters long?
A. 10 centimeters
B. 15 centimeters
C. 20 centimeters
D. 25 centimeters

(3) A rectangular piece has a length of 15 centimeters and a width of 4 centimeters. To estimate its area, which of the following options is closest to the correct value?
A. 50 square centimeters
B. 60 square centimeters
C. 70 square centimeters
D. 80 square centimeters

**2. Mathematical Logical Thinking**

(1) If all X are not Y and all Y are not Z, which of the following statements is true?
A. Some Xs are not Z
B. all X Are Z
C. all Z Are X
D. Some Z are not X

(2) If a person's day is Thursday, what day of the week will it be in three days?
A. Monday
B. Tuesday
C. Wednesday
D. Friday

(3) Solve the following equation: $3y - 4 = 11$. Then the value of y is:
A. 5
B. 6
C. 7
D. 8

**3. Mathematical Intuitive Thinking**
(1) If a rectangle is about 7 meters long and 3 meters wide, the approximate value of its area is:
A. 15 square meters
B. 20 square meters
C. 21 square meters
D. 25 square meters

(2) The square of a number is 144. Which number is the closest to this number?
A. 10
B. 11
C. 12
D. 13

(3) A small bag holds about 100 grams of candy. If there are 4 such bags of candy, the total weight is closest to:
A. 300 grams
B. 350 grams
C. 400 grams
D. 450 grams

**Post-Test Items**
**1. Mathematical Representational Thinking**
(1) For geometric figure identification, the formula for calculating the area of which of the following figures is $A = 1/2 \times \text{base} \times \text{height}$?
A. square
B. rectangle
C. triangle
D. circle

(2) Figure Transformation: Which attribute of a figure does not change after rotating a square 90 degrees?
A. area
B. angle
C. Side length
D. all of the above

(3) Graph area calculation: a rectangle with a length of 10 cm and a width of 5 cm has an area of () square centimeters.
A. 10
B. 5
C. 50
D. 25

(4) Volume Calculation: A square with a side length of 4 cm has a volume of (    ) cubic centimeters.
A.16
B.64
C.8
D.12

(5) A gardener needs to plant flowers in a garden 12 meters long and 8 meters wide. If he needs 20 flowers per square meter, then he needs a total of ( ) flowers.
A.240
B.160
C.96
D.1920

**2. Mathematical Logical Thinking**
(1) If all A's are B's and all B's are C's, which of the following statements is true?
A. all A's are C's
B. all C's are A's
C. Some A's are not C's
D. no A is a C

(2) Algebraic expression simplification: Simplify the following algebraic expression: 3x + 5 - 2x + 7
A. x + 12
B. x + 10
C. 5x + 12
D. x + 7

(3) Equation Solving, Solve the equation 2x + 7 = 15, x = (      )
A. 3
B. 8
C. 4
D. 6

(4) In a class, 60% of the students are girls and 40% are boys. If there are 30 students in the class, the number of girls is (      ).
A.18
B.12
C.10
D.8

(5) If the two angles of a triangle are 50 and 60 degrees, then the degree of the third angle is ( ) degrees.
A.60
B.70
C.80
D.90

**3. Mathematical Intuitive Thinking**
(1) If a circle has a diameter of about 10 centimeters, the area of the circle is closest to:
A. 78 square centimeters
B. 100 square centimeters
C. 314 square centimeters
D. 150 square centimeters

(2) A number divided by 2 gives a result of 15, then the number is:
A. 30
B. 25
C. 20
D. 10

(3) A number is 7. What is its square? ( )
A. 49
B. 14
C. 21
D. 15

(4) A rectangle has a length of 6 cm, a width of 4 cm, and a height of 3 cm. Using an intuitive estimate, the volume of this rectangle is approximately ( ) cubic centimeters.
A.24
B.18
C.12
D.70

(5) The area of a sheet of paper is approximately the area of an A4 sheet of paper. If a school bag can hold 50 such sheets of paper, intuitively estimate the capacity of the bag to be about ( ) cubic centimeters.
A.100
B.200
C.300
D.400

**Table 1: IOC Test**

| | Expert1 | Expert2 | Expert3 | IOC Result |
|---|---|---|---|---|
| Q1 | 1 | 1 | 1 | 1 |
| Q2 | 1 | 1 | 1 | 1 |
| Q3 | 1 | 1 | 0 | 0.667 |
| Q4 | 1 | 1 | 1 | 1 |
| Q5 | 1 | 1 | 0 | 0.667 |
| Q6 | 1 | 1 | 1 | 1 |
| Q7 | 1 | 1 | 1 | 1 |
| Q8 | 1 | 1 | 1 | 1 |
| Q9 | 1 | 1 | 1 | 1 |
| Q10 | 1 | 1 | 1 | 1 |
| Q11 | 1 | 1 | 1 | 1 |
| Q12 | 1 | 1 | 1 | 1 |
| Q13 | 1 | 1 | 1 | 1 |
| Q14 | 1 | 1 | 1 | 1 |
| Q15 | 1 | 1 | 1 | 1 |
| Q16 | 1 | 1 | 1 | 1 |
| Q17 | 1 | 0 | 1 | 0.667 |
| Q18 | 1 | 1 | 1 | 1 |
| Q19 | 1 | 0 | 1 | 0.667 |
| Q20 | 1 | 1 | 1 | 1 |
| Q21 | 1 | 1 | 1 | 1 |
| Q22 | 1 | 1 | 1 | 1 |
| Q23 | 1 | 1 | 1 | 1 |
| Q24 | 0 | 1 | 1 | 0.667 |
| Q25 | 1 | 1 | 1 | 1 |
| Q26 | 1 | 1 | 1 | 1 |
| Q27 | 1 | 1 | 1 | 1 |
| Q28 | 1 | 1 | 1 | 1 |
| Q29 | 1 | 0 | 1 | 0.667 |
| Q30 | 1 | 1 | 1 | 1 |

## Table 2: Reliability Statistics

| Dimension | Cronbach's Alpha | N of Items |
|---|---|---|
| Questioning | 0.902 | 5 |
| Investigation and Research | 0.892 | 5 |
| Analysis and Interpretation | 0.869 | 5 |
| Conclusion Formation | 0.841 | 5 |
| Reflection and Evaluation | 0.887 | 5 |
| Sharing and Communication | 0.908 | 5 |

## Table 3: KMO and Bartlett's Test

| Kaiser-Meyer-Olkin Measure of Sampling Adequacy. | | 0.856 |
|---|---|---|
| Bartlett's Test of Sphericity | Approx. Chi-Square | 1815.360 |
| | df | 435 |
| | Sig. | 0.000 |

## Table 4: Total Variance Explained

| Component | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | Rotation Sums of Squared Loadings | |
|---|---|---|---|---|---|---|---|
| | Total | % of Variance | Cumulative % | % of Variance | Cumulative % | % of Variance | Cumulative % |
| 1 | 11.821 | 39.402 | 39.402 | 39.402 | 39.402 | 12.690 | 12.690 |
| 2 | 3.397 | 11.323 | 50.725 | 11.323 | 50.725 | 12.685 | 25.375 |
| 3 | 1.979 | 6.597 | 57.322 | 6.597 | 57.322 | 12.447 | 37.821 |
| 4 | 1.648 | 5.494 | 62.817 | 5.494 | 62.817 | 11.880 | 49.701 |
| 5 | 1.348 | 4.495 | 67.311 | 4.495 | 67.311 | 11.163 | 60.864 |
| 6 | 1.154 | 3.848 | 71.159 | 3.848 | 71.159 | 10.295 | 71.159 |
| 7 | 0.795 | 2.651 | 73.810 | | | | |
| 8 | 0.744 | 2.481 | 76.291 | | | | |
| 9 | 0.731 | 2.436 | 78.727 | | | | |
| 10 | 0.667 | 2.223 | 80.950 | | | | |
| 11 | 0.631 | 2.103 | 83.053 | | | | |
| 12 | 0.547 | 1.822 | 84.875 | | | | |
| 13 | 0.534 | 1.779 | 86.654 | | | | |
| 14 | 0.439 | 1.463 | 88.117 | | | | |
| 15 | 0.403 | 1.344 | 89.461 | | | | |
| 16 | 0.369 | 1.231 | 90.693 | | | | |

| Component | Initial Eigenvalues | | | Extraction Sums of Squared Loadings | | Rotation Sums of Squared Loadings | |
|---|---|---|---|---|---|---|---|
| | Total | % of Variance | Cumulative % | % of Variance | Cumulative % | % of Variance | Cumulative % |
| 17 | 0.357 | 1.191 | 91.883 | | | | |
| 18 | 0.318 | 1.062 | 92.945 | | | | |
| 19 | 0.310 | 1.034 | 93.978 | | | | |
| 20 | 0.272 | 0.908 | 94.886 | | | | |
| 21 | 0.243 | 0.811 | 95.698 | | | | |
| 22 | 0.213 | 0.710 | 96.408 | | | | |
| 23 | 0.206 | 0.687 | 97.094 | | | | |
| 24 | 0.182 | 0.606 | 97.700 | | | | |
| 25 | 0.164 | 0.548 | 98.248 | | | | |
| 26 | 0.139 | 0.465 | 98.713 | | | | |
| 27 | 0.130 | 0.432 | 99.145 | | | | |
| 28 | 0.101 | 0.338 | 99.483 | | | | |
| 29 | 0.089 | 0.295 | 99.778 | | | | |
| 30 | 0.067 | 0.222 | 100.000 | | | | |

## Table 5: Rotated Component Matrix

| | Component | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| Q1 | 0.171 | 0.070 | 0.176 | **0.791** | 0.277 | 0.183 |
| Q2 | 0.205 | 0.006 | 0.259 | **0.664** | 0.329 | 0.127 |
| Q3 | 0.253 | 0.133 | 0.255 | **0.740** | 0.171 | 0.228 |
| Q4 | 0.043 | 0.115 | 0.246 | **0.806** | -0.002 | 0.231 |
| Q5 | 0.063 | 0.114 | 0.244 | **0.684** | 0.369 | 0.153 |
| Q6 | 0.319 | 0.220 | **0.749** | 0.179 | 0.176 | 0.069 |
| Q7 | 0.198 | 0.144 | **0.716** | 0.244 | 0.254 | 0.038 |
| Q8 | 0.234 | 0.217 | **0.676** | 0.178 | 0.296 | 0.121 |
| Q9 | 0.084 | 0.005 | **0.765** | 0.255 | 0.259 | 0.063 |
| Q10 | -0.023 | 0.163 | **0.771** | 0.220 | 0.123 | 0.182 |
| Q11 | 0.176 | 0.200 | 0.204 | 0.287 | **0.721** | 0.251 |
| Q12 | -0.069 | 0.143 | 0.302 | 0.183 | **0.674** | 0.237 |
| Q13 | 0.159 | 0.025 | 0.426 | 0.265 | **0.550** | 0.208 |
| Q14 | 0.137 | 0.119 | 0.157 | 0.170 | **0.698** | 0.188 |
| Q15 | 0.160 | 0.003 | 0.231 | 0.131 | **0.739** | 0.187 |
| Q16 | 0.180 | 0.080 | 0.035 | 0.248 | 0.161 | **0.723** |
| Q17 | 0.088 | 0.287 | 0.120 | 0.230 | 0.351 | **0.573** |

| | Component | | | | | |
|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** |
| Q18 | 0.155 | 0.144 | 0.275 | 0.089 | 0.136 | **0.762** |
| Q19 | 0.278 | 0.071 | 0.116 | 0.272 | 0.108 | **0.682** |
| Q20 | -0.029 | 0.051 | -0.009 | 0.076 | 0.271 | **0.749** |
| Q21 | 0.307 | **0.716** | 0.069 | 0.095 | 0.151 | 0.111 |
| Q22 | 0.240 | **0.743** | 0.163 | 0.167 | -0.037 | 0.196 |
| Q23 | 0.201 | **0.802** | 0.254 | -0.047 | -0.009 | 0.119 |
| Q24 | 0.293 | **0.749** | -0.002 | 0.153 | 0.220 | -0.014 |
| Q25 | 0.134 | **0.859** | 0.123 | 0.045 | 0.103 | 0.081 |
| Q26 | **0.729** | 0.364 | 0.068 | 0.084 | 0.171 | 0.155 |
| Q27 | **0.822** | 0.237 | 0.041 | 0.162 | 0.092 | 0.099 |
| Q28 | **0.670** | 0.378 | 0.217 | -0.004 | 0.193 | 0.170 |
| Q29 | **0.810** | 0.167 | 0.222 | 0.208 | -0.004 | 0.087 |
| Q30 | **0.778** | 0.229 | 0.174 | 0.146 | 0.110 | 0.143 |

**Table 6 Items, Mean, Standard Deviation, and Interpretation of Satisfaction**

| **Dimension** | **Items** | **Mean** | **Std. Deviation** | **Interpretation** |
|---|---|---|---|---|
| | 1. Do you think that asking questions during the inquiry process is helpful for learning? | 3.55 | 1.204 | High |
| | 2. Were you satisfied that you had enough opportunities to ask questions? | 3.45 | 1.271 | Moderate |
| | 3. Do you feel that your teacher's feedback on the questions you asked was satisfactory? | 3.51 | 1.155 | High |
| | 4. Did you feel clear in your thinking during the questioning process? | 3.36 | 1.073 | Moderate |
| Questioning | 5. Were you able to effectively translate questions into learning objectives? | 3.38 | 1.129 | Moderate |

| Dimension | Items | Mean | Std. Deviation | Interpretation |
|---|---|---|---|---|
| | Total | 3.45 | 1.166 | Moderate |
| | 6. Were you satisfied that the research study helped you understand the learning content more deeply? | 3.56 | 1.154 | High |
| | 7. Were you satisfied with the resources provided by the research study? | 3.62 | 0.984 | High |
| | 8. Were you satisfied with the tasks during the research study? | 3.71 | 1.115 | High |
| | 9. Were you able to effectively organize and document the results of the research study? | 3.58 | 1.306 | High |
| | 10. Were you satisfied with the collaboration that took place during the research study? | 3.51 | 1.205 | High |
| Investigation and Research | Total | 3.60 | 1.153 | High |
| | 11. Were you satisfied with the clarity of the data analysis process? | 3.53 | 1.308 | High |
| | 12. Were you able to effectively extract useful information from the data? | 3.52 | 1.215 | High |
| | 13. Were you able to relate the results of the analysis to the content of the study? | 3.30 | 1.128 | Moderate |
| | 14. Were you satisfied with the task of analyzing and interpreting? | 3.63 | 1.107 | High |
| Analysis and Interpretation | 15. Were you satisfied with the support provided by your teacher during the analysis and interpretation stage? | 3.52 | 1.215 | High |

| Dimension | Items | Mean | Std. Deviation | Interpretation |
|---|---|---|---|---|
| | Total | 3.50 | 1.195 | Moderate |
| | 16. Were you able to form clear conclusions based on the results of the analysis? | 3.71 | 1.027 | High |
| | 17. Were you satisfied with the process of forming conclusions? | 3.74 | 1.042 | High |
| | 18. Were you able to relate the conclusions effectively to the original question? | 3.81 | 1.306 | High |
| | 19. Were you satisfied with the conclusions formed? | 3.50 | 1.166 | Moderate |
| | 20. Are you satisfied with the resolution of difficulties encountered during the conclusion formation stage? | 3.66 | 1.144 | High |
| Conclusion Formation | Total | 3.69 | 1.137 | High |
| | 21. Do you engage in regular self-reflection to assess the effectiveness of your learning? | 3.71 | 0.893 | High |
| | 22. Were you able to identify the challenges you encountered in the learning process? | 3.73 | 0.999 | High |
| | 23. Are you satisfied with the contribution of the reflection and assessment process to improving your learning? | 4.06 | 1.192 | High |
| | 24. Are you satisfied with the feedback on reflection and assessment provided by your teachers? | 3.73 | 0.818 | High |
| Reflection and Evaluation | 25. Are you satisfied with the significance of the | 3.94 | 1.211 | High |

| Dimension | Items | Mean | Std. Deviation | Interpretation |
|---|---|---|---|---|
| | reflection and assessment process? | | | |
| | Total | 3.83 | 1.023 | High |
| | 26. Are you satisfied with how the process of sharing your learning has helped your learning? | 3.71 | 1.126 | High |
| | 27. Were you able to articulate your learning outcomes and conclusions clearly? | 3.43 | 1.153 | High |
| | 28. Do you feel that communication with your classmates helped you to understand your learning? | 3.64 | 1.116 | High |
| | 29. Were you satisfied with the opportunities for sharing and communication? | 3.60 | 1.249 | High |
| | 30. Are you satisfied that you gained new insights from your classmates' sharing? | 3.66 | 1.242 | High |
| Sharing and Communication | Total | 3.61 | 1.177 | High |